B.Sc. (Part-III) (CBCS Pattern) Semester-VI

021A - (DSE-VI) - Mathematics Paper-II : Complex Analysis and Vector Calculus

P. Pages : 2 **GUG/W/24/13361**

Time : Three Hours * 5 0 8 1 * Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) Prove that a necessary condition that $w = f(z) = u(x, y) + iv(x, y)$ be analytic in a region D is that u and v satisfy the Cauchy-Riemann equations $u_x = v_y$ and $u_y = -v_x$. **6**

b) If the function $f(z) = u + i\nu$ be analytic in the domain D, then prove that the families of curves $u(x, y) = c_1$ and $v(x, y) = c_2$ form an orthogonal system, where c_1 and c_2 are arbitrary constants. **6**

OR

c) Prove that $u = y^3 - 3x^2y$ is harmonic. Determine its harmonic conjugate and find the corresponding analytic function f(z). **6**

d) Prove that every bilinear transformation with two non-infinite fixed points α, β is of the form **6**

$$\frac{w-\alpha}{w-\beta} = k\frac{z-\alpha}{z-\beta}$$

Where k is constant.

UNIT – II

2. a) Let f(z) = 2 and (be any curve joining the points z = a and z = b. Then show that **6**

$$\int_C f(z)\,dz = \frac{1}{2}(b^2 - a^2).$$

b) Evaluate $\int_{1-i}^{2+i} (2x + iy + 1)\,dz$ along the straight line joining the points (1 – i) & (2 + i). **6**

OR

c) Evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$, where C is the circle $|z| = \frac{3}{2}$. **6**

d) Find the residue of $f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$ at all its poles in the finite plane. **6**

UNIT – III

3. a) If $\bar{r} = \bar{a}\cos\omega t + \bar{b}\sin\omega t$, show that $\bar{r} \times \dot{\bar{r}} = \omega\bar{a} \times \bar{b}$ and $\ddot{\bar{r}} = -\omega^2\bar{r}$, where $\bar{a}, \bar{b}$ and ω are constant. **6**

b) If $\overline{f} = x^2 2\overline{i} - 2y^3z^2\overline{j} + xy^2z\overline{k}$, find $\operatorname{div}\overline{f}$ and $\operatorname{curl}\overline{f}$ at (1, –1, 1). **6**

OR

c) Prove that $\overline{\nabla} \circ (\overline{f} \times \overline{g}) = (\overline{\nabla} \times \overline{f}) \circ \overline{g} - \overline{f} \circ (\overline{\nabla} \times \overline{g})$ where $\overline{f}, \overline{g}$ are vector point differentiable functions in some region of space. **6**

d) Compute the line integral $\int_C y^2 dx - x^2 dy$ about the triangle whose vertices are (1, 0), (0, 1) & (–1, 0). **6**

UNIT – IV

4. a) Apply Green's theorem to prove that the area enclosed by a simple plane curve C is $\frac{1}{2}\oint_C (x\,dy - y\,dx)$. **6**

b) Evaluate $\iint_S \overline{f} \circ \hat{n}\, dS$ where $\overline{f} = x^2\overline{i} + y^2\overline{j} + z^2\overline{k}$ and S is the surface of the solid cut off by the plane x + y + z = a from the first octant. **6**

OR

c) By expressing $\iint_S \left(y^2z^2\overline{i} + z^2x^2\overline{j} + z^2y^2\overline{k}\right) d\overline{S}$ as a volume integral, evaluate if over the upper part of the sphere $x^2 + y^2 + z^2 = 1$, above the xy – plane. **6**

d) State and prove Stoke's theorem. **6**

5. Solve **any six.**

a) Write Cauchy – Riemann equations in polar form. **2**

b) If $w = f(z) = u + iv$ be analytic in D then prove that $\frac{dw}{dz} = \frac{\partial w}{\partial x}$. **2**

c) Show that $\int_C \frac{1}{2} dz = z\pi i$, where C is | z | = r. **2**

d) Write Cauchy's integral theorem for a simply connected domain D. **2**

e) If $\overline{r} = t\overline{i} + \sin t\,\overline{j} + (t^2 - 1)\overline{k}$, find $\dot{\overline{r}}, \ddot{\overline{r}}$. **2**

f) If $\phi = 3x^2y - y^3z^2$ find grad ϕ at the point (1, –2, –1). **2**

g) State Green's theorem. **2**

h) State Divergence theorem. **2**

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