



- Notes : 1. Solve **all five** questions.  
2. Each question carry equal marks.

**UNIT-I**

1. A) How many ways are there to arrange the seven letters in the word SYSTEMS? In how many of these arrangements do the three Ss appear consecutively? **10**
- B) How many integer solutions are there to the equation  $x_1 + x_2 + x_3 + x_4 = 12$ , with  $x_i \geq 0$ ? **10**  
How many solutions with  $x_i \geq 1$ ? How many solutions with  $x_1 \geq 2, x_2 \geq 2, x_3 \geq 4, x_4 \geq 0$ ?

**OR**

- C) a) How many 5-card hands (subsets) can be formed from a standard 52-card deck? **10**  
b) If a 5-card hand is chosen at random, what is the probability of obtaining a flush (all five cards in the hand are in the same suit)?  
c) What is the probability of obtaining three, but not four, Aces?
- D) Nine students, three from Ms. A's class, three from Mr. B's class, and three from Ms. C's class, have bought a block of 9 seats for their school's homecoming game. If three seats are randomly selected for each class from the nine seats in a row, what is the probability that the three A students, three B students, and three C students will each get a block of three consecutive seats? **10**

**UNIT-II**

2. A) Find the coefficient of  $x^{16}$  in **10**  
 $(x + x^2 + x^3 + x^4 + x^5)(x^2 + x^3 + x^4 + \dots)^5$
- B) How many ways are there to select 25 toys from seven types of toys with between one and six of each type? **10**

**OR**

- C) How many ways are there to distribute 25 identical balls into seven distinct boxes if the first box can have no more than 10 balls but any number can go into each of the other six boxes? **10**
- D) Find the coefficient of  $x^{11}$  in  $x^2(1-x)^{-10}$ . What is the coefficient of  $x^r$ ? **10**

### UNIT-III

3. A) Solve the recurrence relation  $a_n = a_{n-2}$  with  $a_0 = a_1 = 1$ . 10
- B) Solve the recurrence relation  $a_n = a_{n-1} + 3(n-1)$  with  $a_0 = 1$ . 10

OR

- C) Solve the recurrence relation  $a_n = a_{n-1} + n(n-1)$  with  $a_0 = 3$ . 10
- D) Solve the recurrence relation  $a_n = a_{n-1} + a_{n-2}$  with  $a_0 = a_1 = 1$ . 10

### UNIT-IV

4. A) If a school has 80 students with 40 taking French, 40 taking Latin, and 40 taking German, 20 students are taking any given pair of languages, and 10 students are taking all three languages, then how many students are taking no language? 10
- B) How many positive integers  $\leq 120$  are relatively prime to 120. 10

OR

- C) How many ways are there to select a 6-card hand from a regular 52-card deck such that the hand contains at least one card in each suit? How many 6-card hands with a void in at least one suit? 10
- D) Let  $A_1, A_2, \dots, A_n$ , be  $n$  sets in a universe  $\mathcal{U}$  of  $N$  elements. Let  $S_k$  denote the sum of the sizes of all  $k$ -tuple intersections of the  $A_i$ 's. Then show that, 10

$$N(\overline{A_1} \cap \overline{A_2} \cap \dots \cap \overline{A_n}) = N - S_1 + S_2 - S_3 + \dots + (-1)^k S_k + \dots + (-1)^n S_n$$

5. Solve all the four questions.
- A) How many four digit numbers can be formed without repetition of any digit? 5
- B) Find the generating function for  $a_r$ , the number of ways to select  $r$  objects chosen from four types of objects with repetition of up to four objects of each type. 5
- C) A bank pays 4 percent interest each year on money in savings accounts. Find recurrence relations for the amounts of money a person would have after  $n$  years if he follows the investment strategies of 5
- a) Investing 10000 Rs. and leaving it in the bank for  $n$  years.
- b) Investing 1000 Rs. at the end of each year.
- D) How many positive integers  $\leq 50$  are relatively prime to 50? 5

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