B.E. / B.Tech. (Instrumentation Engineering) Model Curriculum Semester-III

IN301 - Applied Mathematics-III (Probability and Statistics)

P. Pages : 2 | GUG/W/24/13906A

Time : Three Hours | Max. Marks : 80

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Notes :
1. All questions carry equal marks.
2. All questions are compulsory.
3. Non programmable calculator is permitted.

1. a) State & prove the property of Laplace Transform of Derivative. **8**

b) Express $f(t)=\begin{cases} t^2, & 0<t<2 \\ 4t, & t>2 \end{cases}$ in terms of unit step function & hence find its Laplace Transform. **8**

OR

2. a) Find Laplace Transform of the function **8**

$$f(t)=\begin{cases} \sin\omega t, & 0<t<\pi/\omega \\ 0 \quad , & \pi/\omega<t<2\pi/\omega \end{cases}$$

Where, $f(t)=f(t+2\pi/\omega)$

b) Show that $\int_0^\infty e^{-t}\frac{\sin^2 t}{t}dt=\frac{1}{4}\log 5$. **8**

3. a) Find $L^{-1}\left[\frac{s^2-6}{(s^3+4s^2+3s)}\right]$. **8**

b) Show that **8**

$$L^{-1}\left[\log\left(\frac{s^2+1}{s(s+1)}\right)\right]=\frac{1}{t}\left[1+e^{-t}-2\cos t\right]$$

OR

4. a) Using Convolution Theorem find $L^{-1}\left[\frac{1}{s^3(s^2+1)}\right]$. **8**

b) Solve $\frac{d^2y}{dt^2}+4\frac{dy}{dt}+5y=5u(t)$, where $u(t)$ is the unit step function given that, $y(0)=1,\ y'(0)=2$ **8**

5. a) Find the Fourier Sine integral of **8**

$$f(x)=\begin{cases} \pi/2\ ; & 0<x<\pi \\ 0\ ; & x>\pi \end{cases}$$

b) Evaluate the $\int_0^\infty \frac{dt}{(a^2+t^2)(b^2+t^2)}$ by using Parseval's Identity. **8**

OR

6. a) Find the Fourier Transform of **8**

$$f(x)=\begin{cases}1-x^2, & |x|\le 1\\ 0\quad , & |x|>1\end{cases}$$

& hence show that $\int_0^{\infty}\frac{\sin x - x\cos x}{x^3}\cdot\cos\frac{x}{2}\,dx=\frac{3\pi}{16}$

b) Find $f(x)$ if its Fourier sine transform is $\frac{e^{-a\lambda}}{\lambda}$. **8**

7. a) i) Solve $\frac{\partial^2 z}{\partial x\cdot\partial y}=\sin x\cdot\sin y$, for which $\frac{\partial z}{\partial y}=-2\sin y$ when $x=0$ & $z=0$ when y is an odd multiple of $\pi/2$. **4**

ii) Form an partial differential equation by eliminating arbitrary function from $Z=f(x+at)+g(x-at)$. **4**

b) Solve $x(y^2-z^2)\frac{\partial z}{\partial x}+y(z^2-x^2)\frac{\partial z}{\partial y}=z(x^2-y^2)$ **8**

OR

8. a) Solve the partial differential equation by method of separation of variables $\frac{\partial u}{\partial x}=4\frac{\partial u}{\partial y}$, given $u(0,y)=8e^{-3y}$. **8**

b) Solve $y^2p-xyq=x(z-2y)$. **8**

9. a) Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix}2&2&1\\1&3&1\\1&2&2\end{bmatrix}$. Hence find A^{-1}. **8**

b) Using Sylvester's theorem verify that $\sin^2 A+\cos^2 A=I$, where $A=\begin{bmatrix}2&4\\3&1\end{bmatrix}$. **8**

OR

10. a) Find the value of K for which the equations **8**

$$2x+(4-k)y=-7$$
$$(2-k)x+2y=-3$$
$$2x+5y=k-6$$

are consistent. Find the general solution for these value of K.

b) Use matrix method to solve the differential equation **8**

$$\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0,\ y(0)=1, y'(0)=2$$

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