B.E. / B.Tech. Electronics & Communication / Telecommunication Engineering
(Model Curriculum) Semester-IV

SE202 - Probability, Random Process & Numerical Method

P. Pages : 3 **GUG/W/24/13912**

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry equal marks.
2. Assume suitable data wherever necessary.
3. Use of Non-programmable calculator is permitted.

1. a) The joint probability function of two discrete random variable X and Y is given by **8**

$$f(x,y)=\begin{cases} cxy, & x=1,2,3 \ \& \ y=1,2,3 \\ 0, & \text{otherwise} \end{cases}$$

Find (i) The constant C (ii) $P(X=3, Y=1)$ (iii) $P(1 \le X \le 2, \ Y \le 3)$ (iv) find the conditional probability function of X given that Y = 2.

b) The chance that a doctor A will diagnose a disease X correctly is 60%. The chance that a patient will die by his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of doctor A, who had disease X died. What is the chance that his disease was diagnosed correctly? **8**

OR

2. a) If X is a binomial random variable with mean 4 and variance = 2.4, find the distribution of X. **8**

b) If X is a Poisson variable such that $P(X=2)=P(X=4)+15\,P(X=6)$ **8**

Find (i) λ (ii) standard deviation of X (iii) third moment of X (iv) β_2.

3. a) An urn holds 5 white and 3 black marbles. If two marbles are drawn at random without replacement and X denotes the number of white marbles (i) find the probability function and (ii) the distribution function. **8**

b) A random variable X has density function **8**

$$f(x)=\begin{cases} Kx^2, & 1 \le x \le 2 \\ Kx, & 2 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

Find constant 'K' and the distribution function.

OR

4. a) A random variable X has density function given by **8**

$$f(x)=\begin{cases}2e^{-2x}, & x\ge 0\\ 0 \quad , & x<0\end{cases}$$

Find (i) E(x) (ii) Var(X) (iii) $E[(x-1)^2]$ (iv) first four moments about the origin.

b) Find moment generating function and first four moments about the origin for r.v. X given by $X=\begin{cases}1\ , & \text{prob } 1/2\\ -1, & \text{prob } 1/2\end{cases}$ **8**

5. a) Let $f(x,y)=\begin{cases}2e^{-(x+2y)}, & x\ge 0,\ y\ge 0\\ 0 & , x<0,\ y<0\end{cases}$ **8**

be the joint density function of random variable X and Y. Find (i) conditional expectation of X given Y and (ii) conditional variance of X given Y.

b) Let X and Y be random variables having joint density function **8**

$$f(x,y)=\begin{cases}e^{-(x+y)}, & x\ge 0,\ y\ge 0\\ 0 & , \text{otherwise}\end{cases}$$

Find (i) Var(X) (ii) Var(Y) (iii) σ_x (iv) σ_y.

OR

6. a) Every year, a man trades his car for a new car. If he has a Maruti, he trades it for an Ambassador. If he has an Ambassador, he trades it for a Santro. However, if he has a Santro, he is just as likely to trade it for a new Santro as to trade it for a Maruti or an Ambassador. In 2000 he bought his first car, which was a Santro. Find the probability that he has (i) 2002 Santro (ii) 2003 Ambassador by Markov chain. **8**

b) Apply Chebyshev's theorem to calculate (i) $P(5<X<15)$ (ii) $P(|X-10|\ge 3)$ for a random variable X with mean $\mu=10$ and variance $\sigma^2=4$. **8**

7. a) Verify central limit theorem in the case where $X_1, X_2, ---X_n$ are independent and identically distributed with Poisson distribution. **8**

b) Prove that convergence with probability-I implies convergence in probability. **8**

OR

8. a) Verify central limit theorem for a random variable X which is Binomially distributed with mean np and standard deviation $\sqrt{npq}$. **8**

b) Let X(n) be a martingale sequence on $n\ge 0$, satisfying $E\left\{X^2(n)\right\}\le c<\infty$ for all n for some C. Then $X(n)\to X$ as $n\to\infty$, where X is the limiting random variable. **8**

9. a) The mean square derivative of a WSS random process X(t) exists at time t if the autocorrelation $R_{XX}(\tau)$ has derivatives up to order two at $\tau = 0$. **8**

b) Let the input to the state equation $\dot{X}(t) = A\ X(t) + B\ W(t)$ be the white Gaussian process W(t). Then the output X(t) is a vector Gauss-Markov random process. **8**

OR

10. a) The transfer function of an LSI system is given by **8**

$H(w) = \operatorname{sgn}(w)\left(\frac{w}{2\pi}\right)^2 \exp\left[-j\left(w \cdot \frac{8}{\pi}\right)\right] W(w)$, where $\operatorname{sgn}(w)$ is the algebraic sign function and where $W(w) = \begin{cases} 1\ , & \text{for } |w| \le 40\pi \\ 0\ , & \text{else} \end{cases}$

compute the average measurable power in the band 0 to 1 Hertz.

b) Let $X(n) \to X$ and $Y(n) \to Y$ in the m.s. sense with $E\left[|X|^2\right] < \infty$ and $E\left[|Y|^2\right] < \infty$ then show that **8**

i) $\lim\limits_{n\to\infty} E\left[|X(n)|^2\right] = E\left[|X|^2\right]$

ii) $\lim\limits_{n\to\infty} E\left[X(n) \cdot Y(n)\right] = E[XY]$

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