M.Sc. - I (Mathematics) (NEP Pattern) Semester-I

NEP-62 - Major DSC-2 – Topology

P. Pages : 2 | GUG/W/24/15113

Time : Three Hours | Max. Marks : 80

* 5 9 9 6 *

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. A) Prove that the set of all real numbers is uncountable. **8**

B) Prove that the set of all rational numbers is denumerable. **8**

OR

C) Prove that $2^{\aleph_0} = C$. **8**

D) Prove that every infinite set is equipotent to a proper subset of itself. **8**

UNIT – II

2. A) For any set E in a topological space (X, $\mathfrak{I}$), prove that $i(E) = \left[c(E^c)\right]^c$. **8**

B) If A, B are subsets of the topological space (X, $\mathfrak{I}$), then prove that the derived set has the following properties: **8**

i) If $A \subseteq B$, then $d(A) \subseteq d(B)$

ii) $d(A \cup B) = d(A) \cup d(B)$

OR

C) Let X = {a, b, c} & let $T = \{\phi, \{a\}, \{b, c\}, X\}$. Find the derived sets of following subsets of X: **8**

i) {a, b} ii) {b}

D) For any set E in a topological space (X, $\mathfrak{I}$), Prove that $c(E) = E \cup d(E)$. **8**

UNIT – III

3. A) Prove that the union E of any family $\{C_\lambda\}$ of connected sets having a nonempty intersection is a connected set. **8**

B) Prove that a topological space (X, $\mathfrak{I}$) is compact iff any family of closed sets having the finite intersection property has a nonempty intersection. **8**

OR

C) If f is a one-to-one continuous mapping of $(X, \mathfrak{I})$ into $(X^*, \mathfrak{I}^*)$, then prove that f maps every dense-in-itself subset of X onto a dense-in-itself subset of X^*. **8**

D) Prove that a mapping f of X into X^* is open iff $f(i(E)) \subseteq i^*(f(E))$ for every $E \subseteq X$. **8**

UNIT – IV

4. A) Prove that a topological space X is a T_0 - space iff the closures of distinct points are distinct. **8**

B) Prove that every compact subset E of a Hausdorff space X is closed. **8**

OR

C) Prove that a topological space X is a T_1-space iff every subset consisting of exactly one point is closed. **8**

D) Prove that in a Hausdorff space, a convergent sequence has a unique limit. **8**

5. A) Define: **4**
i) Equipotent sets ii) Denumerable set

B) Define : **4**
i) Topology ii) Limit point

C) Define : **4**
i) Connected set ii) Compact set

D) Define : **4**
i) T_0 - space ii) T_1 - space
iii) T_2 - space iv) First axiom space

****