M.Sc. (Mathematics) (New CBCS Pattern) Semester-I

PSCMTH02 - Real Analysis-I

P. Pages : 2 | GUG/W/24/13738

Time : Three Hours | Max. Marks : 100

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Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT - I

1. a) State & prove the Stone-Weierstrass theorem. **10**

b) Let α be monotonically increasing on [a, b]. Suppose $f_n \in \Re(\alpha)$ on [a, b], for h = 1, 2, 3, ----, and suppose $f_n \to f$ uniformly on [a, b]. Then prove that $f \in \Re(\alpha)$ on [a, b], and $\int_a^b F d\alpha = \lim_{h\to\infty} \int_a^b f_n\, d\alpha$. **10**

OR

c) Suppose $f_n \to f$ uniformly on a set E in a metric space. Let x be a limit point of E, and suppose that $\lim_{t\to x} f_n(t) = A_n\ (n = 1, 2, 3....)$. Then prove that $\{A_n\}$ converges, and $\lim_{t\to x} f(t) = \lim_{n\to\infty} A_n$. **10**

d) If K is compact, if $f_n \in \tau(K)$ for n = 1, 2, 3,….., and if $\{f_n\}$ is pointwise bounded & equicontinuous on K, then prove that. **10**

a) $\{f_n\}$ is uniformly bounded on K,

b) $\{f_n\}$ contains a uniformly convergent subsequence.

UNIT - II

2. a) Suppose f maps an open set $E \subset IR^n$ into IR^m. Then prove that $f \in \tau'(E)$ iff the partial derivatives $D_j F_i$ exist and are continuous on E for $1 \le i \le m,\ 1 \le j \le n$. **10**

b) State & prove the inverse function theorem. **10**

OR

c) If X is a complete metric space, and if ϕ is a contraction of X into X, then prove that there exists one & only one $x \in X$ such that $\phi(x) = x$. **10**

d) Suppose E is an open set in IR^n, f maps E into IR^m, f is differentiable at $x_0 EE$, g maps an open set containing F (E) into IR^k, and g is differentiable at $f(x_0)$. Then prove that the mapping F of E into IR^k defined by $F(x) = g(f(x))$ is differentiable at x_0, and $F'(x_0) = g'(f(x_0)) f'(x_0)$. **10**

UNIT - III

3. a) Prove that any atlas $u = \{\cup_\alpha, \phi_\alpha\}$ on a locally Euclidean space is contained in a unique maximal atlas. **10**

b) Consider S^1 as the unit circle in the real plane IR^2 with defining equation $x^2 + y^2 = 1$, and describe a $\not\subset^\infty$ atlas with four charts on it. **10**

OR

c) Let $\{(\cup_\alpha, \phi_\alpha)\}$ be an atlas on a locally Euclidean space. If two charts (v, ψ) and (ω, σ) are both compatible with the atlas $\{(\cup_\alpha, \phi_\alpha)\}$, then prove that they are compatible with each other. **10**

d) Define – (i) Locally Euclidean space of dimension n (ii) Coordinate neighbourhood (iii) $\not\subset^\infty$ - compatible charts. **10**

UNIT - IV

4. a) Define Lie group and show that GL (n, IR) is a Lie group. **10**

b) State and prove the inverse function theorem for manifolds. **10**

OR

c) If $(\cup, \phi)$ is a chart on a manifold M of dimension n, then prove that the co-ordinate map $\phi : \cup \to \phi(\cup) \subset IR^n$ is a diffeomorphism. **10**

d) Let M & N be manifolds and $\Pi : M \times N \to M$, $\pi(p, q) = p$ the projection to the first factor. Prove that Π is a $\subset^\infty$ map. **10**

5. a) Define pointwise convergence & uniform convergence for sequence of functions. **5**

b) Define- **5**
i) Contraction mapping and ii) Differentiable function

c) Define- **5**
i) Topological manifold ii) Smooth manifold.

d) Define- **5**
i) Diffeomorphism ii) Smooth functions on a manifold

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