M.Sc. - Second Year (Mathematics) (New CBCS Pattern) Semester-III

PSCMTH11 - Complex Analysis

P. Pages : 2

Time : Three Hours

GUG/W/24/13755

Max. Marks : 100

Notes :
1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) Prove that when a limit of a function f(z) exists at point z_0, it is unique. **10**

b) Prove that if $f'(z)=0$ everywhere in a domain D, Then F(z) must be constant Throughout D. **10**

OR

c) If a function $f(z)=u(x,y)+iv(x,y)$ is analytic in a domain D. Then prove its component functions u and v are harmonic in D. **10**

d) Show that- **10**

i) $ex\left(\frac{2+\pi i}{4}\right)=\sqrt{e/2}(1+i)$ ii) $\log(i)^3 \neq 3\log i$

UNIT – II

2. a) State and prove the maximum module principle. **10**

b) State and prove the Laurent's Theorem of the series representation as- **10**

$$f(z)=\sum_{n=0}^{\infty} a_n (z-z_0)^n + \sum_{n=1}^{\infty} \frac{b_n}{(z-z_0)^n}$$

OR

c) Let F be analytic everywhere inside and on a simple closed contour C_1 taken in the positive sense. If z_0 is any positive interior to C, then Prove that- **10**

$$f(z_0)=\frac{1}{2\pi i}\int_c \frac{f(z)dz}{z-z_0}$$

d) Prove that if a power series $\sum_{n=0}^{\infty} a_n (z-z_0)^n$ converges when $z=z_1 (z_1 \neq z_0)$ then it is absolutely at each point z in the open disk. **10**

$|z-z_0|<R_1$ where $R_1=|z_1-z_0|$

UNIT – III

3. a) Evaluate the improper integral. $\int_0^\infty \frac{x \sin 2x}{x^2+3} dx$ **10**

b) State and prove the Rouche's Theorem. **10**

OR

c) Evaluate $\int_0^\infty \frac{x^2}{x^6+1} dx$ **10**

d) Let C be simple closed contour described in the positive sense. If a function f is analytic inside and on C except for a finite number of singular points. z_k inside C Then show that- **10**

$$\int_c f(z) dz = 2\pi i \sum_{k=1}^{n} \underset{z=z_k}{\text{Res}} \left(f_{(z)}\right)$$

UNIT – IV

4. a) Explain the transformation $w = \frac{1}{z}$ and show that $\lim_{z \to z_0} T(z) = T(z_0)$ **10**

b) Find the special case of the linear fractional. Transformations that maps the points $z_1 = -1,\ z_2 = 0$ and $z_3 = 1$ on the points $w = -i,\ w_2 = 1_1, w_3 = i$ **10**

OR

c) Show that the Transformation $w = \log\left[\frac{z-1}{z+1}\right]$ transforms the plane u>0 onto the strip $0 < v < \pi$. **10**

d) Show that the image of the vertical strip $0 \le x \le 1,\ y \ge 0$ under the mapping $w = z^2$ is a closed semi parabolic region. **10**

5. a) State the sufficient condition for the differentiability. **5**

b) Show that $\int_0^{\pi/4} e^{it} dt = 1/\sqrt{2} + i\left(1 - \frac{1}{\sqrt{2}}\right)$ **5**

c) Find the pole and Residue of- $f(z) = \frac{z^3+2z}{(z-i)^3}$ **5**

d) Find the fixed points of the transformations. $w = \frac{z-1}{z+1}$ **5**

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