



- Notes :
1. All questions carry equal marks.
  2. All questions are compulsory.
  3. Non programable calculator is permitted.

1. a) Evaluate  $\lim_{x \rightarrow 0} \frac{x^y - y^x}{x^x - y^y}$ . 4
- b) Find a and b such that  $\lim_{x \rightarrow \infty} \frac{x(1 + a \cos x) - b \sin x}{x^3} = 1$ . 4
- c) Prove that  $\log(\sec x + \tan x) = x + \frac{x^3}{6} + \frac{x^5}{24} + \dots$ . 8

**OR**

2. a) If  $y = \left[ x + \sqrt{1 + x^2} \right]^m$  then prove that 8
- $$(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$$
- b) Obtain Taylor's expansion of  $\tan^{-1} x$  in power of  $x - \frac{\pi}{4}$  up to the third-degree terms. 8
3. a) Find the value of n that the equation  $v = r^n [3 \cos^2 \theta - 1]$  satisfies the 8
- Relation  $\frac{\partial}{\partial r} \left( r^2 \frac{\partial v}{\partial r} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial v}{\partial \theta} \right) = 0$ .
- b) If  $u = \log \left( \frac{x^2 + y^2}{\sqrt{x} + \sqrt{y}} \right)$  find the value of 8
- i)  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$
- ii)  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$

**OR**

4. a) Prove that if  $z$  is a homogeneous function of  $x, y$  of order  $n$  and if  $z = f(u)$  then  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = n \frac{f(u)}{f'(u)}$  8

- b) If  $\theta = t^n e^{-\frac{r^2}{4t}}$  find what value of  $n$  will make  $\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \theta}{\partial r} \right) = \frac{\partial \theta}{\partial t}$ . 8

5. a) If  $x = \sqrt{uw}, y = \sqrt{vw}, z = \sqrt{uv}$  and  $u = r \sin \theta \cos \phi, v = r \sin \theta \sin \phi, w = r \cos \theta$  then find  $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$ . 8

- b) If  $u = xy + yz + zx, v = x + y + z, w = x^2 + y^2 + z^2$  are functionally related? If so, find the relation between them. 8

**OR**

6. a) If  $u = \frac{x+y}{1-xy}$  and  $v = \tan^{-1} x + \tan^{-1} y$ , find  $\frac{\partial(u, v)}{\partial(x, y)}$  are  $u$  and  $v$  functionally related? If so, find the relation between them. 8

- b) Find the minimum value of  $x^2 + y^2 + z^2$  given  $ax + by + cz = p$ . 8

7. a) Using the differentiation under the integral sign. Prove that  $\int_0^1 x^p (\log x)^n dx = \frac{(-1)^p n!}{(n+1)^{n+1}}$  8

- b) Find the volume of the solid formed by revolution of the curve  $(a-x)y^2 = a^2x$  about its asymptote. 8

**OR**

8. a) Show that  $\int_0^{\frac{\pi}{2}} \frac{\sin^{2m-1} x \cos^{2n-1} x}{(a \sin^2 x + b \cos^2 x)^{m+n}} dx = \frac{1}{2a^m b^n} \beta(m, n)$  8

- b) By differentiating under the integral sign, evaluate  $\int_0^a \frac{\log(1 + \sin a \cos x)}{\cos x} dx$  8

9. a) Fit the  $y = ax^2 + b$  for the following data 8

x	12	16	20	24	26	30
y	6.44	7.5	6.9	10.76	11.76	14.0

- b) Fit an equation of the form  $y = ab^x$  to the following data.

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x	2	3	4	5	6
y	144	172.3	207.4	248.8	298.5

**OR**

10. a) Two lines of regression are given by  $8x - 10y + 66 = 0$  and  $40x - 18y = 214$ .

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If  $\sigma_x^2 = 9$  Find

- i) mean values of x and y
- ii) the standard deviation of y
- iii) the coefficient of correlation between x and y

- b) Fit the curve  $y = ax + \frac{b}{x}$  to the following data:

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x	1	2	3	4	5	6	7	8
y	5.43	6.28	8.23	10.32	12.63	14.86	17.27	19.51

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