M.Sc. - I (Mathematics) (New CBCS Pattern) Semester-I

PSCMTH05(A) - Numerical Analysis

P. Pages : 3 **GUG/W/24/13741**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve **all five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) Discuss the Regula Falsi method for finding the roots of equation. **10**

b) Let $f(x), f'(x)$ and $f''(x)$ are continuous for all x in some interval containing of α and assume $f'(a) \neq 0$, then prove that if x_0 and x_1 are chosen sufficiently close to α the iterates x_n of $x_{n+1} = x_n - f(x_n) \cdot \dfrac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}$, $n \geq 1$ will converges to α the order of convergence will be $P = \dfrac{1+\sqrt{5}}{2} \simeq 1.62$. **10**

OR

c) Assume $f(x), f'(x)$ and $f''(x)$ are continuous for all x in some neighbourhood of α and assume $f(\alpha) = 0,\ f'(\alpha) \neq 0$, then prove that if x_0 is chosen sufficiently close to α the iterates $x_n, n \geq 0$ of $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$, will converges to α. **10**

d) Obtain the iteration formula of Muller's method for finding roots of polynomial. **10**

UNIT – II

2. a) Let $x_0, \ldots\ldots, x_n$ be distinct real numbers, and let f(x) be n times continuously differentiable on the interval $H\{x_0, \ldots\ldots, x_n\}$, Then show that **10**

$$f[x_0, \ldots\ldots, x_n] = \int \ldots\ldots \int_{T_n} f^n(t_0 x_0 + \ldots. + t_n x_n) dt_1 \ldots. dt_n$$

$$T_n = \left\{ (t_1, t_2, \ldots\ldots, t_n) \middle| t_1 \geq 0, \ldots. t_n \geq 0, \sum_1^n t_i \leq 1 \right\},\ t_0 = 1 - \sum_1^n t_i .$$

d) Prove that for $k \geq 0$ $f[x_0, x_1, \ldots\ldots, x_k] = \dfrac{1}{k! h^k} \Delta^k f_0$, where $f_0 = f(x_0)$ & $f_i = f(x_i)$. **10**

OR

c) Let $x_1, x_2, \ldots\ldots, x_n$ be distinct real numbers, and let f be a given real valued function with $n+1$ continuous derivatives on the interval $I_t = H\{t, x_0, x_1 \ldots\ldots, x_n\}$, with t some given real number. Then prove that there exists $\xi \in I_t$ with **10**

$$f(t) - \sum_{j=0}^{n} f(x_j) l_j(t) = \frac{(t-x_0)\ldots\ldots(t-x_n)}{(n-1)!} f^{(n+1)}(\xi)$$

d) Find the Hermite interpolating polynomial for which- **10**

$p(a) = f(a) \quad p'(a) = f'(a)$

$p(b) = f(b) \quad p'(b) = f'(b)$

UNIT – III

3. a) Let f(x) be continuous for $a \le x \le b$ and let $\in > 0$. Then prove that there is a polynomial p(x) for which **10**

$|f(x) - p(x)| < \in, \ a \le x \le b$

b) Discuss the Gram-Schmidt theorem. **10**

OR

c) Let $\{\phi_n(x) \mid n \ge 0\}$ be an orthogonal family of polynomials on (a, b) with weight function w(x). With such a family we always assume implicitly that degree $\phi_n = n \ n \ge 0$. If f(x) is a polynomial of degree m, then prove that **10**

$$f(x) = \sum_{n=0}^{m} \frac{(f, \phi_n)}{(\phi_n, \phi_n)} \phi_n(x)$$

d) Prove that for $f, g \in C[a, b]$ $|(f, g)| \le \|f\|_2 \|g\|_2$ **10**

UNIT – IV

4. a) Obtain simple Simpson's rule of integration. Obtain error estimate. **10**

b) Obtain simple trapezoidal rule with error. **10**

OR

c) Assume [a, b] is finite. Then prove that the error in Gaussian quadrature. **10**

$E_n(f) = \int_a^b w(x) f(x) dx \sum_{j=1}^{n} w_j f(x_j)$ Satisfies

$|E_n(f)| \le 2\left[\int_a^b w(x) dx\right] \rho_{2n-1}(f) \ n \ge 1$ with $\rho_{2n-1}(f)$ the minimax error from

$\rho_n(f) = \underset{\deg(q) \le n}{\text{Infimum}} \|f - q\|_\infty$

d) Obtain the expression for Peano-Kernel error formula. **10**

5. a) Show that the Newton's method for determining a square root of A has the form. **5**

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{A}{x_n}\right).$$

b) Obtain the expression for $p_1(x)$ and $p_2(x)$ by Lagrange's interpolation. **5**

c) Prove that for $f, g \in C[a, b]$ $\|f + g\| \leq \|f\|_2 + \|g\|_2$ **5**

d) Define: **5**

i) Asymptotic error estimate

ii) Degree of precision

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