B.Sc. (Part-I) (CBCS Pattern) Semester-II

USMT-04 - Mathematics Paper-II - Partial Differential Equations

P. Pages : 2 **GUG/W/24/11587**

Time : Three Hours * 4 5 3 9 * Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT-I

1. a) Obtain the partial differential equation which represents the set of all spheres whose centres lie along the 3-axis. **6**

b) Solve $\frac{dx}{x(y^2-z^2)} = \frac{dy}{-y(z^2+x^2)} = \frac{dz}{z(x^2+y^2)}$ **6**

OR

c) Solve total differential equation **6**

$y(1+z^2)\,dx - x(1+z^2)\,dy + (x^2+y^2)\,dz = 0$

d) If u is a function of x, y, z which satisfies the PDE **6**

$(y-z)u_x + (z-x)u_y + (x-y)u_z = 0$

Show that u contains x, y, z only in combination $x+y+z$ and $x^2+y^2+z^2$.

UNIT-II

2. a) Show that the partial differential equation $z = px + qy$ is compatible with any equation $f(x, y, z, p, q) = 0$, where f is homogeneous in x, y, z. **6**

b) Solve $z^2(1+p^2+q^2) = k^2$ **6**

OR

c) Solve $p^2 + q^2 = x^2 + y^2$ **6**

d) Find the complete integral of the equation $z^2 = pqxy$, by using Charpit's method. **6**

UNIT-III

3. a) Prove that a solution of partial differential equation $(D - mD')z = 0$ has the form $z = F(y + mx)$, where F is an arbitrary function. **6**

b) Solve the equation **6**

$$\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial y^3} = e^{x+2y}$$

OR

c) Solve the equation **6**

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \cos x \cos 2y$$

d) Solve the equation **6**

$$(D^2 - 2DD' - 8D'^2)\, z = \sqrt{2x + 3y}$$

UNIT-IV

4. a) Prove that the solution of non-homogeneous differential equation $(aD + bD' + c)\, z = 0$ is **6**

$z = e^{-cx/a} F(ay - bx)$

b) Solve $D(D - 2D' - 3)\, z = e^{x+2y}$ **6**

OR

c) Solve the equation **6**

$$x^2 \frac{\partial^2 z}{\partial x^2} - 4xy \frac{\partial^2 z}{\partial x \partial y} + 4y^2 \frac{\partial^2 z}{\partial y^2} + 6y \frac{\partial z}{\partial y} = x^3 y^4$$

d) Reduce the equation $r = x^2 t$ to canonical form. **6**

5. Solve **any six.**

a) Solve **2**

$(yz + xyz)dx + (2x + xyz)dy + (xy + xyz)dz = 0$

b) Obtain the partial differential equation by eliminating arbitrary functions of **2**
$z = f(x - y)$.

c) Write condition of compatibility. **2**

d) Find the complete solution of $f(z, p, q) = 0$ **2**

e) Solve $\left(\frac{\partial^4 z}{\partial x^4} - \frac{\partial^4 z}{\partial y^4} \right) z = 0$. **2**

f) Find particular integral of $(2D - 3D')z = e^{x-y}$ **2**

g) Solve the equation $(D + 2D' - 3)z = 0$ **2**

h) Solve the equation $(D^2 - D')z = 0$ **2**

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