M.Sc.(Mathematics) (New CBCS Pattern) Semester-I

PSCMTH03 - Topology-I

P. Pages : 2 **GUG/W/24/13739**

Time : Three Hours * 5 4 1 6 * Max. Marks : 100

Notes : 1. Solve **all five** questions.
2. Each question carries equal marks.

UNIT - I

1. a) Prove that the union of a denumerable number of denumerable sets is a denumerable set. **10**

b) Prove that $2^a > a$ for every cardinal number a. **10**

OR

c) Prove that the set of all real numbers is uncountable. **10**

b) Prove that the set of all rational numbers is denumerable. **10**

UNIT - II

2. a) Prove that $\mathfrak{I}^*$ is a topology for X^*. **10**

b) Let $X = \langle a, b, c \rangle$ & let $\mathfrak{I} = \langle \phi, \{a\}, \{b\}, \{a, b\}, X \rangle$. Find derived sets of following subsets of X: **10**

i) {b} ii) {a, c}

iii) {a, b}

OR

c) Prove that a set F is a closed subset of a topological space (X, $\mathfrak{I}$) iff its complement is an open subset of the space. **10**

d) Let X be any infinite set, and let J be the family consisting of ϕ & all complements of finite sets show that $\mathfrak{I}$ is a topology for X. **10**

UNIT - III

3. a) If E is a subset of a subspace (X*, $\mathfrak{I}^*$) of a topological space (X, $\mathfrak{I}$), then prove that E is $\mathfrak{I}^*$- connected iff it is $\mathfrak{I}$-connected. **10**

b) If f is a homeomorphism of X onto X*, then prove that f maps every isolated subset of X onto an isolated subset of X*. **10**

OR

c) If f is a continuous mapping of $(X, \mathfrak{I})$ into $(X^*, \mathfrak{I}^*)$, then prove that f maps every compact subset of X onto a compact subset of X*. **10**

d) If C is a connected subset of a topological space $(X, \mathfrak{I})$ which has a separation $X=A|B$, then prove that either $C \subseteq A$ or $C \subseteq B$. **10**

UNIT - IV

4. a) Prove that every compact subset E of a Hausdorff space X is closed. **10**

b) Prove that in a second axiom space, every collection of nonempty disjoint, open sets is countable. **10**

OR

c) Prove that a topological space X is a $T_{\perp}$ - space iff every subset consisting of exactly one point is closed. **10**

d) Prove that a topological space X is normal iff for any closed set F and open set G containing F, there exists an open set G* such that $F \subseteq G^*$ and $C(G^*) \subseteq G$. **10**

5. a) Define equipotent sets & cardinal numbers. **5**

b) Define closure of a set and interior of a set. **5**

c) Define dense-in-itself subset & isolated subset of a topological space. **5**

d) Define first axiom space and second axiom space. **5**

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