M.Sc. (Mathematics) (NEP Pattern) Semester-I

NEP-64-3 / DSE3 - Ordinary Differential Equations

P. Pages : 2 **GUG/W/24/15117**

Time : Three Hours *6000* Max. Marks : 80

Notes : 1. All questions are compulsory.
2. Each questions carry equal marks.

UNIT – I

1. a) Prove that for any real x_0, and constants α, β there exists a solution ϕ of the initial value problem $L(y) = 0,\ y(x_0) = \alpha,\ y'(x_0) = \beta$ on $-\infty < x \infty$. **8**

b) Let ϕ_1, ϕ_2 be any two linearly independent solutions of L(y) = 0 on an interval I. Then prove that every solution ϕ of L(y) = 0 can be written uniquely as $\phi = c_1\phi_1 + c_2\phi_2$, where c_1, c_2 are constant. **8**

OR

c) If ϕ_1, ϕ_2 are two solutions of L(y) = 0 on an interval I containing a point x_0, then prove that $w(\phi_1, \phi_2)(x) = e^{-\phi_1(x-x_0)} w(\phi_1, \phi_2)(x_0)$ **8**

d) Explain the method to solve a non-homogeneous equation of order two. **8**

UNIT – II

2. a) Prove that there exist n linearly independent solution of a linear L(y) = 0 on I. **8**

b) Prove that If $\phi_1, ..., \phi_n$ are n solutions of L(y) = 0 on an interval I, they are linearly independent there if and only if, $w(\phi_1, ..., \phi_n)\ (z) \neq 0$ for all x in I. **8**

OR

c) Solve the Euler's equation of second order. **8**

d) Discuss the Bessel equation. **8**

UNIT – III

3. a) Let g, h be continuous real valued functions for $a \le x \le b, c \le y \le d$ respectively, and consider the equation $h(y)y' = g(x) - (1)$. **8**

If G, H are any functions such that $G' = g,\ H' = h$ and C is any constant such that the relation $H(y) = G(x) + C$, defines a real-valued differentiable function ϕ for x in some interval I contained in $a \le x \le b$, then prove that ϕ will be a solution of equ^n(1) on I. Conversely, If ϕ is a solution of equ^n(1) on I. It satisfies the relation H(y) = G(x) + C on I, for some constant C.

b) Prove that a function ϕ is a solution of the initial value problem $y' = f(x, y),\ y(x_0) = y_0$ on an interval I if and only if it is a solution of the integral equation $y = y_0 + \int_{x_0}^{x} f(t, y)\,dt$ on I. **8**

OR

c) Let f be a continuous real-valued function on the rectangle $R : |x - x_0| \le a,\ |y - y_0| \le b$, $(a,\ b > 0)$ & Let $|f(x, y)| \leqq m$ for all (x, y) in R. further suppose that f satisfies a Lipschitz condition with constant K in R Then prove that the successive approximations $\phi_0(x) = y_0,\ \phi_{k+1}(x) = y_0 + \int_{x_0}^{x} f(t,\ \phi_k(t))\,dt\ (k = 0, 1, 2, ...)$, converges on the interval $I : |x - x_3| \leqq \alpha = \min\{a, b/m\}$ to a solution ϕ of the initial value problem $y' = f(x, y),\ y(x_0) = y_0$ on I. **8**

d) Let f, g be continuous on R, and suppose f satisfies a Lipschitz condition there with Lipschitz constant k. let ϕ and ψ be solution of $y' = f(x, y),\ y(x_0) = y_1$ and $y' = g(x, y),\ y(x_0) = y_2$ resp. on an interval I containing x_0, with graphs contained in R. If the inequalities $|f(x, y) - g(x, y)| \le \in ((x, y) \text{ in } R)$ and $|y_1 - y_2| \leqq \delta$ are valid, then prove that $|\phi(x) - \psi(x)| \leqq \delta e^{K|x - x_0|} + \frac{\in}{K}(e^{k|x - x_0|} - 1)$ for all x in I. **8**

UNIT – IV

4. a) Describe existence and uniqueness for linear systems. **8**

b) Let $a_1, ..., a_n, b$ be continuous complex-valued functions on an interval I containing a point x_0. If $a_1, ..., a_n$ are any n constants. Then prove that there exists one, and only one, solution ϕ of the equation $y^{(n)} + a_1(x)y^{(n-1)} + ... + a_n(x)y = b(x)$ on I satisfying $\phi(x_0) = a_1,\ \phi'(x_0) = \phi_2, ..., \phi^{(n-1)}(x_0) = d_n$. **8**

OR

c) Describe complex n-dimensional space and write dawn the properties of magnitude and distance. **8**

d) Describe existence and uniqueness of solution of system. **8**

5. Solve the following.

a) Define linear equations of the first order. **2**

b) Verify that the function $\phi_1(x) = x^3 (x > 0)$ satisfies the equation $x^2 y'' - 7xy' + 15y = 0$. **4**

c) Describe exact equations. **4**

d) Solve the equation $xy'' - 2y' = x^3$. **4**

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