B.Sc.- I (NEP Pattern) Semester-I

BSCMA501 - DSC - Mathematics : Differential Calculus

P. Pages : 3 **GUG/W/24/15924(S)**

Time : Three Hours Max. Marks : 80

* 7 6 7 9 *

Notes : 1. Solve all the **five** questions.
2. First four questions carry equal 15 marks and the fifth question carry 20 marks.

UNIT – I

1. Solve **any three**. **5**

a) Prove that if $\lim_{x \to x_0} f(x)$ exists then it is unique.

b) Show that the function f defined by **5**

$$f(x) = x.\sin\frac{1}{x}, \; x \neq 0$$
$$= 0 \quad , \text{otherwise}$$

is continuous at x = 0

c) Verify the Rolle's theorem for the function $f(x) = x^2 + x - 6$ in [-3, 2] **5**

d) Prove that if a real valued function f defined on [a, b] is continuous in [a, b] and differentiable in (a, b) then there is at least one point $c \in (a, b)$ such that $f'(c) \cdot (b - a) = f(b) - f(a)$. **5**

e) Verify Cauchy mean value theorem for $f(x) = e^x$, $g(x) = e^{-x}$ in [a, b] **5**

UNIT – II

2. Solve **any three**.

a) Obtain Maclaurin's series for f(x) = sin x. **5**

b) Show that $e^{x\cos x} = 1 + x + \frac{x^2}{2} - \frac{x^3}{3} + ...$ **5**

c) Expand sin x in powers of $\left(x - \frac{\pi}{2}\right)$. **5**

d) Prove that if limit of a function f(x, y) as (x, y) $\to (x_0, y_0)$ exists, then it is unique. **5**

e) Prove that the function $f(x, y) = x + y$ is continuous $\forall (x, y) \in R^2$. **5**

UNIT – III

3. Solve **any three**.

a) If $y = \log(ax+b)$ then show that $y_n = (-1)^{n-1}(n-1)!a^n(ax+b)^{-n}$ and hence find y_{10} at x = 1, for $y = \log(3x-2)$. **5**

b) If $y^3 + 3ax^2 + x^3 = 0$ then show that $y^5 y_2 + 2a^2x^2 = 0$. **5**

c) Find y_n, for f(x) = $\tan^{-1}\frac{x}{a}$. **5**

d) If $y = (x + \sqrt{1+x^2})^m$ then show that- **5**

i) $(1+x^2)y_2 + xy_1 - m^2y = 0$

ii) $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$

e) If $y = (x^2-1)^n$ then prove that $(x^2-1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$. **5**

UNIT – IV

4. Solve **any three**.

a) If $u = 3(ax+by+cz)^2 - (x^2+y^2+z^2)$ and $a^2+b^2+c^2 = 1$ then show that **5**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$$

b) Prove that if z = f(x, y) has differential at (x_0, y_0) then f is continuous at (x_0, y_0). **5**

c) If $u = f(x, y)$ be a homogeneous function of degree n in x, y then prove that **5**

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial x} = nu.$$

d) If $x = r\sin\theta\cos\varnothing, y = r\sin\theta\sin\varnothing, x = r\cos\theta$ then find $\frac{\partial(x,y,z)}{\partial(r,\theta,\varnothing)}$ **5**

e) Find the stationary points of $x^4 + y^4 - 2x^2 + 4xy - 2y^2$. **5**

5. Solve **any ten**.

a) Write $\in - \delta$ definition of continuity of a function. **2**

b) Evaluate $\lim\limits_{x\to 3}(2x^3 - 3x^2 + 7x - 11)$. **2**

c) State the Rolle's theorem. **2**

d) State the Taylor's theorem for f(x+h). **2**

e) Write the $\in - \delta$ definition of limit of function of two variables. **2**

f) Using algebra of limits, prove that $\lim_{(x,y)\to(1,2)} (x^2 + xy - 2x - y) = -1$. **2**

g) If $y = e^{-2x}$, find y_{11}. **2**

h) If y = Asinmx + Bcosmx then prove that $y_2 + m^2 y = 0$. **2**

i) State the Leibnitz theorem. **2**

j) Prove that $f_{xy} = f_{yx}$, for $f(x, y) = 2x^3y^2 - 3xy^2 + x - 2y$. **2**

k) State first and second chain rule. **2**

l) Define absolute maxima and absolute minima. **2**

****