M.Sc. (Mathematics) (NEP Pattern) Semester-II

Major Elective DSE-10- Advanced Topics in Topology

P. Pages : 2

Time : Three Hours

GUG/W/24/15402

Max. Marks : 80

Notes : 1. Solve all the **five** questions.
2. Each questions carries equal marks.

UNIT - I

1. a) Prove that every separable metric space is second axiom. **8**

b) Prove that topological space X is completely normal iff every subspace of X is normal. **8**

OR

c) Prove that a normal space is completely regular iff it is regular. **8**

d) Prove that every countably compact matric space is totally bounded. **8**

UNIT - II

2. a) Show that $\Pi_{\lambda}\ X_{\lambda}$ is Housdorff iff each X_{λ} is Housdorff. **8**

b) Show that x & y are topological spaces the family of all sets of the form $U \times V$ with U is open in X & V open in Y is a base for a topology for $X \times Y$. **8**

OR

c) State and prove Tikhonov Theorem. **8**

d) Show that $X \times Y$ is compact iff X & Y are compact. **8**

UNIT - III

3. a) Prove that for every open covering of metric space, there is a locally finite open cover which refines it. **8**

b) Prove that a subset G of Y is open in the quotient topology relatives to $f : X \to Y$ iff $f^{-1}(G)$ is an open subset of X. **8**

OR

c) Show that every paracompact regular space is normal. **8**

d) Prove that every second axiom $T_3-\text{space}$ is metrizable. **8**

UNIT - IV

4. a) Prove that A topological space is Hausdorff iff no filter can converge to more than one point in it. **8**

b) Prove that every filter is contained in an ultrafilter. **8**

OR

c) For a topological space X, prove that the following statements are equivalent. **8**
   i) X is compact.
   ii) Every net in X has a cluster point in X.
   iii) Every net in X has a convergent subnet in X.

d) Show that an ultrafilter converges to a point iff that point is a cluster point of it. **8**

5. a) Prove that every metric space is Hausdorff space. **4**

b) Prove that the projection Π_x and Π_y are continuous and open mapping. **4**

c) If f is a continuous mapping of the topological space X onto the topological space Y, then prove that the topology for Y must be the quotient topology. **4**

d) Define : **4**
   i) Directed set.
   ii) Net

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