

M.Sc. (Mathematics) (NEP Pattern) Semester-II
Major Elective DSE-6 - Combinatorics

P. Pages : 2

Time : Three Hours



GUG/W/24/15398

Max. Marks : 80

- Notes : 1. Solve all **five** questions.
2. Each question carry equal marks.

UNIT - I

1. a) How many ways are there to form a three letter sequence using the letters a, b, c, d, e, f 8
i) With repetition of letters allowed?
ii) Without repetition of any letter?
iii) Without repetition and containing the letter e?
- b) A committee of k people is to be chosen from a set of seven women and four men. How 8
many ways are there to form the committee if
i) The committee consists of three women and two men?
ii) The committee can be any positive size but must have equal numbers of women and men?
iii) The committee has four people and one of them must be Mr. Baggins?

OR

- c) How many arrangements of the seven letters in the word SYSTEMS have the E occurring 8
somewhere before the M? How many arrangements have the E somewhere before the M and the three Ss grouped consecutively?
- d) How many integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 12$, with 8
 $x_i \geq 0$? How many solutions with $x_i \geq 1$? How many solutions with
 $x_1 \geq 2, x_2 \geq 2, x_3 \geq 4, x_4 \geq 0$?

UNIT - II

2. a) Find the coefficient of x^{16} in $(x^2 + x^3 + x^4 + \dots)^5$. What is the coefficient of x^r ? 8
- b) How many ways are there to select 25 toys from seven types of toys with between two and 8
six of each type?

OR

- c) Use generating functions to find the number of ways to collect 15 Rs. from 20 distinct 8
people if each of the first 19 people can give a rupee or nothing and the twentieth person can given either 1 Rs. or 5 Rs. or nothing.

- d) Find the coefficient of x^{47} in $(x^{10} + x^{11} + \dots + x^{25})(x + x^2 + \dots + x^{15})(x^{20} + x^{21} + \dots + x^{45})$. 8

UNIT - III

3. a) Solve the recurrence relation $a_n = 2a_{n-1} + 3a_{n-2}$ with $a_0 = a_1 = 1$. 8
- b) Solve the recurrence relation $a_n = 3a_{n-1} - 4n + 3 \times 2^n$ to find its general solution. Also find the solution when $a_1 = 8$. 8

OR

- c) Solve the recurrence relation $a_n = 2a_{n-1} + 1$ with $a_1 = 1$. 8
- d) Solve the recurrence relation $a_n = 3a_{n-1} + 4a_{n-2}$ with $a_0 = a_1 = 1$. 8

UNIT - IV

4. a) If a school has 100 students with 40 taking French, 40 taking Latin, and 40 taking German, 20 students are taking any given pair of languages, and 10 students are taking all three languages, then how many students are taking no language? 8

- b) Let A_1, A_2, \dots, A_n , be n sets in a universe \mathcal{U} of N elements. Let S_k denote the sum of the sizes of all k -tuple intersections of the A_i s. Then 8
- $$N(\overline{A_1} \cap \overline{A_2} \cap \dots \cap \overline{A_n}) = N - S_1 + S_2 - S_3 + \dots + (-1)^k S_k + \dots + (-1)^n S_n.$$

OR

- c) How many ways are there to select a 6-card hand from a regular 52-card deck such that the hand contains at least one card in each suit? How many 6-card hands with a void in at least one suit? 8
- d) How many positive integers ≤ 70 are relatively prime to 70. 8

5. Solve all the **four** questions.
- a) There are five different Spanish books, six different French books, and eight different Transylvanian books. How many ways are there to pick an unordered pair of two books not both in the same language? 4
- b) Find the generating function for a_r , the number of ways to select r balls from three green, three white, three blue, and three gold balls. 4
- c) Find a recurrence relation for the number of ways to arrange n distinct objects in a row. Find the number of arrangements of eight objects. 4
- d) If a school has 100 students with 50 students taking French, 40 students taking Latin, and 20 students taking both languages, how many students take no language? 4
