B.E. / B.Tech. (Model Curriculum) Semester-II

BSC104 / ESC104 - Engineering Mathematics-II

P. Pages : 3 GUG/W/24/13173

Time : Three Hours Max. Marks : 80

Notes : 1. All questions carry equal marks.
2. Use of non-programmable calculator is permitted.

1. a) Solve the differential equations $\sin x \frac{dy}{dx} + 2y = \tan^3\left(\frac{x}{2}\right)$. **8**

b) Solve $\sec^2 y \frac{dy}{dx} + x \frac{1}{\cos^2 y} 2\sin y.\cos y = x^3$ **8**

OR

2. a) Solve the differential equations **4**

$$\left(1+e^{\frac{x}{y}}\right)dx + \left(1-\frac{x}{y}\right)e^{\frac{x}{y}}.dy = 0$$

b) Solve the differential equation $(D^2+1)\,y = \operatorname{cosec} x$ **4**

c) Solve the differential equation **8**

$$(D^2-1)\,y = e^{-x}.\sin(e^{-x}) + \cos(e^{-x})$$

3. a) Solve the differential equation **8**

$$\frac{d^2y}{dx^2} - y = x\sin x + (1+x^2)\,e^x$$

b) Solve $(x^2D^2 + xD + 1)y = \log x.\sin(\log x)$. **4**

c) The radial displacement in a rotating disc at a distance r from the axis is given by **4**

$r^2 \frac{d^2u}{dr^2} + r\frac{du}{dr} - u + kr^3 = 0,$ where K is a constant solve the equation under the equation under the conditions $u = 0$ when $r = 0$ and $u = 0$ when $r = a$.

OR

4. a) A mechanical system with two degree of freedom satisfies the equations **8**

$$2\frac{d^2x}{dt^2} + 3\frac{dy}{dt} = 4; \quad 2\frac{d^2y}{dt^2} + 3\frac{dy}{dt} = 0$$

Obtain the expressions for x and y interms of t given that $x, y, \frac{dx}{dt}, \frac{dy}{dt}$ all vanish at $t = 0$.

b) Solve $x^2 \dfrac{d^2y}{dx^2} + 9x\dfrac{dy}{dx} + 25y = 50$. **4**

c) Solve $\left(\dfrac{d^2y}{dx^2}\right) = 2(y^3 + y)$, given that $y = 0$, $\dfrac{dy}{dx} = 1$, at $x = 0$. **4**

5. a) Evaluate by changing the order of integration **8**

$$\int_0^{\pi/2}\int_0^{y} \cos 2y.\sqrt{1-k^2\sin^2 x}.dxdy$$

b) Evaluate $\int_0^{e}\int_1^{\log y}\int_1^{e^x} \log z.dz.dx.dy$ **8**

OR

6. a) Find the area lying inside the cardioid $r = a(1+\cos\theta)$ and outside the circle $r = a$. **8**

b) Evaluate $\int_0^{a}\int_0^{a-\sqrt{a^2-y^2}} \dfrac{xy.\log(x+a)}{(x-a)^2} dx\, dy$ **8**

7. a) Show that **8**

i) $(\overline{b} - \overline{c}).[(\overline{c} - \overline{a}) \times (\overline{a} - \overline{b})] = 0$

ii) $[\overline{b} + \overline{c}, \overline{c} + \overline{a}, \overline{a} + \overline{b}] = 2[\overline{a}\,\overline{b}\,\overline{c}]$

b) A particle moves along the curve **8**

$x = t^2 + 1, y = t^2, z = 2t + 5$

where t is the time. Find the component of its velocity and

OR

8. a) Find the directional derivative of $\dfrac{1}{r}$ in the direction of $\overline{r}$, where $\overline{r} = xi + yj + zk$. A particle moves along the curve $\overline{r} = (t^3 - 4t)i + (t^2 + 4t)j + (8t^2 - 3t^3)k$, where t is the time. Find the magnitude of the tangential and normal component f its acceleration at $t = 2$. **8**

b) Show that $\overline{F} = (y^2\cos x + z^3)\hat{i} + (2y\sin x - 4)\hat{j} + (3xz^2 + 2)\hat{k}$ **8**

is a conservative vector field find a function ϕ such that $\hat{F} = \nabla\phi$. Also find the work done in moving the particle from $(0,1,-1)$ to $\left(\dfrac{\pi}{2}, -1, 2\right)$.

9. a) Show that $\overline{E} = \frac{\overline{r}}{r^2}$ is irrotational. Find ϕ such that $\overline{E} = -\nabla\phi$, where $\overline{r} = x\hat{i} + y\hat{j} + z\hat{k}$. **8**

b) Find the total work done in moving a particle in a force field given by $\overline{F} = 3xyi - 5zj + 10xk$ along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$. **8**

OR

10. a) Evaluate $\iint_S \overline{A}.\hat{n}\, ds$, where $\overline{A} = (x + y^2)i - 2xj + 2yzk$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant. **8**

b) Prove that, for every field $\overline{V}$; **8**

i) div curl $\overline{V} = 0$

ii) curl grad $\overline{V} = 0$

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