M.Sc. (Mathematics) (New CBCS Pattern) Semester-II

PSCMTH08 : Advanced Topics in Topology

P. Pages : 2 **GUG/W/24/13748**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all the **five** questions.
2. Each questions carries equal marks.

UNIT- I

1. a) Prove that a Hausdorff space (X, τ) is completely regular iff the family of all cozero sets of real-valued continuous mappings of X is a base for the topology τ. **10**

b) Prove that every Lindelof metric space is second axiom. **10**

OR

c) Prove that every metric space is completely normal. **10**

d) Prove that every separable metric space is second axiom. **10**

UNIT - II

2. a) Prove that $X \times Y$ is compact iff $X \& Y$ are compact. **10**

b) Prove that $X \times Y$ is dense-in-itself iff at least one of the spaces $X \& Y$ is dense-in itself. **10**

OR

c) Prove that $\Pi_\lambda X_\lambda$ is first axiom iff each space X_λ is first axiom and all but a countable number are indiscrete. **10**

d) Prove that the projections Π_X and Π_Y are continuous and open mappings but not necessarily closed, and so the product topology is the smallest topology for which the projections are continuous. **10**

UNIT - III

3. a) Prove that a subset G of Y is open in the quotient topology relative to $F : X \to Y$ iff $F^{-1}(G)$ is an open subset of X. **10**

b) Prove that every second axiom T_3 - space is metrizable. **10**

OR

c) If ξ is a locally finite family of subsets of a topological space X, then prove that the family of closures of members of ξ is also locally finite, and in either case, $C\left(\bigcup\{E : E \in \xi\}\right) = \bigcup\{C(E) : E \in \xi\}$. **10**

d) Prove that every paracompact regular space is normal. **10**

UNIT - IV

4. a) Prove that a topological space is Hausdorff iff limits of all nets in it are unique. **10**

b) Prove that a topological space is Hausdorff iff no filter can converge to more than one point in it. **10**

OR

c) Let $S : D \to X$ be a net in a topological space and let $x \in X$. Then prove that x is a cluster point of S iff there exists a subnet of S which converges to x in X. **10**

d) Prove that every filter is contained in an ultrafilter. **10**

5. a) Define : **5**
- i) Completely regular space.
- ii) Complete normal space.

b) Define Tichonov topology. **5**

c) Define : **5**
- i) Paracompact space.
- ii) Locally Finite Family.

d) Define : **5**
- i) Filter.
- ii) Base for filter.

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