B.Sc.- II (CBCS Pattern) Semester-IV

USMT-08 - Mathematics Paper-II - Elementary Number Theory

P. Pages : 2 **GUG/W/24/12015**

Time : Three Hours * 4 7 5 4 * Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Let a and b be integers such that b > 0. Then prove that there unique integers q and r such that a = bq + r with $0 \le r < b$. **6**

b) Prove that $3 \mid n(n+1)(2n+1), n \in Z$. **6**

OR

c) Find the gcd for 275 and 200 and express it in the form 275x+200y. **6**

d) If $(a,b)=1$, then prove that $(ac,b)=(c,b)$ **6**

UNIT – II

2. a) Prove that every positive integer greater than one has at least one prime divisor. **6**

b) Find the gcd and lcm of a = 18900 and b = 17160 by writing each of the numbers a and b in prime factorization canonical form. **6**

OR

c) Prove that the Format number F_5 is divisible by 641 and hence is composite. **6**

d) Prove that any two distinct fermat numbers relatively prime. **6**

UNIT – III

3. a) Prove that congruence is an equivalence relation. **6**

b) What is the remainder when the sum $1^5 + 2^5 + 3^5 + ... + 200^5$ is divided by 4? **6**

OR

c) Let P be prime. Then prove that the positive integer a is its own inverse modulo p iff $a \equiv 1 (\mod p)$ or $a \equiv -1 (\mod p)$. **6**

d) If m, n are distinct primes, then show that $m^{n-1}+n^{m-1}-1$ is a multiple of mn. **6**

UNIT – IV

4. a) Solve linear congruence $5x=3(\mathrm{mod}\,14)$ by using Euler theorem. **6**

b) Find all the positive-integers x and y such that $x^{\phi(y)}=y$. **6**

OR

c) Prove that Mobius μ-function is multiplicative. **6**

d) Let m be a positive integer and a be an integer with $(a,m)=1$, Then prove that $a^{\phi(m)}\equiv 1(\mathrm{mod}\,m)$. **6**

5. Solve **any six.**

a) Prove that n^3-n is divisible by 6. **2**

b) Prove that $(a,bc)=1$ then $(a,b)=1$ and $(a,c)=1$. **2**

c) Define a fermat numbers. **2**

d) Define linear Diophantine equation. **2**

e) Define a linear congruence. **2**

f) Find the remainder obtained upon dividing the sum $1!+2!+3!+4!+5!+....+1000!+1001!$ by 12. **2**

g) If p is prime and $p \nmid a$, , then show that $a^{p-1}\equiv 1(\mathrm{mod}\,p)$. **2**

h) Define Pythagorean Triple. **2**

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