B.E. / B.Tech. Mechanical Engineering (Model Curriculum) Semester-III

BSC202 - Applied Mathematics-III (PDE, Probability & Statistics)

P. Pages : 3 **GUG/W/24/14056**

Time : Three Hours * 5 5 7 4 * Max. Marks : 80

Notes : 1. All questions carry equal marks.
2. All questions are compulsory.
3. Non programmable calculator is permitted.

1. a) Solve $\left(z^2-2yz-y^2\right)p+\left(xy-zx\right)q=xy-zx$ **8**

b) Solve $4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u$ using method of separation of variable, given $u=3e^{-y}-e^{-5y}$ when x=0. **8**

OR

2. a) Find the Laurent's series expansion of the function $R(z)=\frac{1}{(z-1)(z-2)}$ in the region. **8**

i) $|<|z|<2$

ii) $|z|>2$

b) Find the value of $\oint\frac{(12z-7)}{(z-1)^2(2z+3)}dz$, where C is a circle. **8**

i) |z|=2

ii) $|z+i|=\sqrt{3}$ by using Cauchy's Residue Theorem.

3. a) Consider the experiment of throwing two dice. let x denote the sum on the two dice. Find- **8**

i) Probability mass function

ii) Distribution function

iii) $P(1\le x\le 7)$.

b) Let x & y be continuous random variables having joint density function. **8**

$$f(x,y)=\begin{cases}c\left(x^2+y^2\right); & 0\le x\le 1;0\le y\le 1\\ 0, & \text{otherwise}\end{cases}$$

Find-

i) C ii) $P(x<1/2,y>1/2)$

iii) Marginal distribution function of x & y

OR

4. a) A random variable x has density function. **8**

$$f(x)=\begin{cases}\dfrac{1}{(b-a)}, & a<x<b\\ 0, & \text{otherwise}\end{cases}$$

Find-

i) E(x)
ii) V(x)
iii) The moment generating function
iv) The 1st four moment about origin.

b) A random variable x has the density function given by **8**

$$f(x)=\begin{cases}e^{-x}, & x\geq 0\\ 0, & \text{otherwise}\end{cases}$$

Find the coefficient of

i) Skewness ii) Kurtosis

5. a) **8**

Verify Cayley-Hamilton theorem for matrix $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$ & hence find A^{-1}.

b) **8**

If $A=\begin{bmatrix}2 & 4\\ 3 & 1\end{bmatrix}$, then prove that $\sec^2 A-\tan^2 A=I$. Using Sylvester's theorem.

OR

6. a) Reduce the quadratic form **8**

$8x_1^2+7x_2^2+3x_3^2-12x,\ x_2+4x, x_3-8x_2x_3$

to canonical form by an orthogonal transformation.

b) **8**

Solve $\dfrac{d^2y}{dt^2}-5\dfrac{dy}{dt}+6y=0$, given $y(0)=2,\ y'(0)=5$.

7. a) Find Fourier series for $f(x)=2x-x^2$ in the interval $0<x<2$. **8**

b) Find the Fourier sine transform of $e^{-|x|}$ & hence show that **8**

$$\int_0^{\infty}\frac{x\sin mx}{1+x^2}dx=\frac{\pi}{2}e^{-m}, m>0.$$

OR

8. a) Solve $\left(D^2+2D+5\right)y = e^{-t}\sin t$, given $y(0)=0,\ y'(0)=1$. **8**

b) Find $L^{-1}\left[\dfrac{1}{s^2(s+1)^2}\right]$ by using convolution them. **8**

9. a) Find the real root of $x\log_{10}x - 1.2 = 0$ by Newton-Raphson method correct upto four decimal places. **8**

b) Apply Crout's method to solve the equations **8**
$3x+2y+7z=4,\ 2x+3y+z=5,\ 3x+4y+z=7$

OR

10. a) Solve $x+7y-3z=-22,\ 5x-2y+3z=18$ & $2x-y+6z=22$ by using Gauss-Seidel method. **8**

b) Use Runge-Kutta method to find appropriate value of y for x = 0.2 when $\dfrac{dy}{dx} = xy+y^2$, given $y(0)=1,\ h=0.1$. **8**

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