M.Sc. - II (Mathematics) (NEP Pattern) - Semester-III

03NEPMATH03 - Paper-III : Mathematical Methods

P. Pages : 3 **GUG/W/24/16015**

Time : Three Hours Max. Marks : 80

Notes : 1. Solve all **five** questions.
2. Each questions carry equal marks.

UNIT – I

1. a) Find the Fourier transform of **8**

i) $f(x) = e^{-a|x|}, \quad a > 0$

ii) $f(x) = |x| e^{-a|x|}, \quad a > 0$

b) If C_1 and C_2 are constant, then prove that **8**

i) $F[C_1 f_1(x) + C_2 f_2(x)] = C_1 F[f_1(x)] + C_2 F[f_2(x)]$

ii) $F_s[C_1 f_1(x) + C_2 f_2(x)] = C_1 F_s[f_1(x)] + C_2 F_s[f_2(x)]$

OR

c) Find f(x), if its Fourier sine transform is $\sqrt{\frac{2}{\pi}} \frac{\xi}{1+\xi^2}$. **8**

d) If $F(\xi)$ and $G(\xi)$ are complex Fourier transform of f(x) and g(x) respectively. Then prove that **8**

i) $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\xi)\overline{G(\xi)}\, d\xi = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\overline{g(x)}\, dx$

ii) $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} |F(\xi)|^2\, d\xi = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} |f(x)|^2\, dx$

UNIT – II

2. a) Let f(x) and $f'(x)$ be continuous and $f''(x)$ be sectionally continuous in $0 \le x \le a$. Then prove that **8**

i) $f_c[f''(x); x \to n] = -f'(0)(-1)^n f'(a) - \frac{n^2\pi^2}{a^2} \bar{f}_c(n)$

ii) $f_s[f''(x); x \to n] = \frac{n\pi}{a} f(0) - (-1)^n \frac{n\pi}{a} f(a) - \frac{n^2\pi^2}{a^2} \bar{f}_c(n)$

b) Find the solution of the one dimensional heat conduction equation in a bar with the temperature distribution satisfying $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ under the boundary condition $\frac{\partial u}{\partial x}(x,t) = 0$ and at $x = \pi$ and the given initial condition $u(x,0) = f(x),\ 0 \le x \le \pi$. **8**

OR

c) Solve the diffusion equation **8**

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t},\ 0 < x < a, t > 0$$

For its solution satisfying the boundary condition.

$\frac{\partial u(0,t)}{\partial x} = \frac{\partial u(a,t)}{\partial x} = 0,\ t > 0$ And the initial condition $u(x,0) = f(x),\ 0 \le x \le a$

Using proper finite Fourier transform.

d) A uniform string of length π is stretched between its ends and initially one end, say x = 0, is given a small oscillation a sinwt while the other end is kept fixed. Using finite Fourier transform prove that the displacement of the point x of the string at time t is given by **8**

$$u(x,t) = a \sin wt\ \sin\frac{w(\pi - x)}{c}\ \operatorname{cosec}\frac{w\pi}{c} + \frac{2acw}{\pi}\Sigma_{n=1}^{\infty}\frac{1}{w^2 - n^2c^2}\ \text{sinnx sinnct},$$

If the displacement u(x, t) of the string satisfies the PDE $\frac{\partial^2 u}{\partial t^2} = C^2\frac{\partial^2 u}{\partial x^2}$.

UNIT – III

3. a) If Laplace transform of a function f(t) is $\overline{f}(p)$. The prove that **8**

$$L\left[\int_0^t f(\tau)d\tau\right] = \frac{\overline{f}(p)}{p}$$

b) Let f(t) be a periodic function of period τ, so that $f(t + n\tau) = f(t)$ for n = 1, 2, 3 ……. If f(t) is a piecewise continuous function for t > 0, then prove that **8**

$$L[f(t)] = \frac{1}{1 - e^{-pt}}\int_0^t e^{-pt}\ f(t)dt$$

OR

c) Evaluate $L^{-1}\left[\frac{6p^2 + 22p + 18}{p^3 + 6p^2 + 11p + 6}\right]$ **8**

d) Solve the boundary value problem $\frac{d^2x}{dt^2} + 9x = \cos 2t,$ under the boundary conditions $x(0) = 1,\ x\left(\frac{\pi}{2}\right) = -1$. **8**

UNIT – IV

4. a) If $\overline{f}_n(\xi)$ be Hankel transform of f(r) of order n, then prove that Hankel transform of f(r) of order n is given $H_n\left[f'(r);\xi\right] = \dfrac{\xi}{2n}\left[(n-1)\,\overline{f}_{n+1}(\xi) - (n+1)\,\overline{f}_{n-1}(\xi)\right],\ n \ge 1$ **8**

b) The vibration u(r, t) of a large membrane is given by **8**

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2},\ r \ge 0,\ t \ge 0$$

With the conditions $u(r,0) = f(r)$, $\dfrac{\partial u(r,0)}{\partial t} = g(r)$ find $u(r,t)$ for $t > 0$.

OR

c) If $f^*(s)$ and $g^*(s)$ are Mellin transforms of the functions f(x) and g(x) respectively, then prove that $M^{-1}\left[f^*(s)\,g^*(s); s \to x\right] = \int_0^\infty f\left(\dfrac{x}{t}\right) g(t)\dfrac{dt}{t}$ **8**

d) If $M\left[f(x);\ x \to s\right] = f^*(s)$. Then prove that **8**

i) $M\left[\int_0^x f(t)dt;\ x \to s\right] = -\dfrac{1}{s} f^*(s+1)$

ii) $M\left[\int_0^x dy\left\{\int_0^y f(t)dt\right\};\ x \to s\right] = \dfrac{1}{s(s+1)} f^*(s+2)$

5. a) If $F_s\left[f(x)\right] = F_s\left[\xi\right]$. Then prove that $F_s\left[f(ax)\right] = \dfrac{1}{a}F_s\left(\dfrac{\xi}{a}\right)$. **4**

b) Let f(x) be continuous and $f'(x)$ be sectionally continuous in $0 \le x \le a$. Then prove that $f_s\left[f'(x);\ x \to n\right] = -\dfrac{n\pi}{a}\overline{f}_c(n),\ n \in N$ **4**

c) Find the Laplace transform of cosat. **4**

d) Define: **4**

i) Hankel transform

ii) Mellin transform

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