

B.Tech. / B.E. (Model Curriculum) Semester-I
BSC103 - Engineering Mathematics-I

P. Pages : 3

Time : Three Hours



GUG/W/24/13166

Max. Marks : 80

- Notes : 1. All questions carry equal marks.
2. Use of Non-programmable calculator is permitted.

1. a) If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, prove that $(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$. 8

b) Evaluate 8

i) $\lim_{x \rightarrow 0} \frac{x \cos x - \log(1+x)}{x^2}$

ii) $\lim_{x \rightarrow 0} \frac{e^x - e^{\sin x}}{x - \sin x}$

OR

2. a) Find the n^{th} derivate of $\frac{1}{x^2 + a^2}$. 8

b) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, show that $(y_{n+2})_0 = (n+1)^2 (y_n)_0$ Hence, using Maclaurin's series, 8

prove that $\frac{\sin^{-1} x}{\sqrt{1-x^2}} = x + \frac{2}{3}x^3 + \frac{8}{15}x^5 + \dots$

3. a) If $z = e^{ax+by} \cdot f(ax-by)$, prove that $b \frac{\partial z}{\partial x} + a \frac{\partial z}{\partial y} = 2abz$. 4

b) If $u = \sin^{-1} (x^2 + y^2)^{1/5}$, prove that 6

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{2}{25} \tan u \cdot (2 \sec^2 u - 5).$$

c) If $z = f(u, v)$, $u = e^x \cos y$, $v = e^x \sin y$, show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (u^2 + v^2) \left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} \right)$. 6

OR

4. a) If $\theta = t^n e^{-r^2/4t}$ find what value of n will make $\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \theta}{\partial r} \right) = \frac{\partial \theta}{\partial t}$. 4
- b) If $u = \tan^{-1} \left(\frac{y^2}{x} \right)$, show that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\sin 2u \cdot \sin^2 u$. 6
- c) If $\phi(cx - az, cy - bz) = 0$, show that $ap + bq = c$ where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$. 6
5. a) If $u = \frac{x+y}{x-y}$, $v = \frac{xy}{(x-y)^2}$. Are u and v functionally related? If so, find the relationship. 8
- b) The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. 8

OR

6. a) Expand $e^x \cos y$ in the neighborhood of $(1, \pi/4)$ upto the terms of second degree. 8
- b) An open rectangular tank is to have capacity 256 cu. Meters. Find the dimensions of the tank so that its surface area is minimum. 8
7. a) For $c > 1$, prove that $\int_0^\infty \frac{x^c}{c^x} dx = \frac{\sqrt{c+1}}{(\log c)^{c+1}}$. 4
- b) Evaluate $\int_0^2 x(8-x^3)^{1/3} dx$. 4
- c) By differentiation under the integral sign. Show that $\int_0^\pi \frac{\log(1 + \sin \alpha \cdot \cos x)}{\cos x} dx = \pi \alpha$ 8

OR

8. a) Find the mean value of the distances from one corner of the square to any point in the square of side a. 8
- b) Show that $\int_0^{\pi/2} \frac{1}{\sin \theta} \log \left(\frac{a + b \sin \theta}{a - b \sin \theta} \right) d\theta = \pi \sin^{-1} \left(\frac{b}{a} \right)$; $\left| \frac{b}{a} \right| < 1$ by differentiation under the integral sign. 8

9. a) Find the correlation coefficient between x and y from the values given. Find also the two regression equations. 8

x	1	2	3	4	5	6	7	8	9	10
y	10	12	16	28	25	36	41	49	40	50

- b) Fit a curve $y = ax + bx^2$ for the following data. 8

x	1	2	3	4	5	6
y	2.51	5.82	9.93	14.84	20.55	27.06

OR

10. a) Find the two missing terms in the following data. 8

x	1.0	1.5	2.0	2.5	3.0	3.5
f(x)	0.4401	-	0.5767	0.4971	-	-0.3801

- b) Use Lagrange's interpolation formula to find $f(0.5)$ from the following data. 8

x	0	1.0	1.5	2.5	3.0
f(x)	1.0	0.77	0.51	-0.05	-0.26
