B.Sc. - III (CBCS Pattern) Semester-VI

021C - (DSE-VIII) Mathematics Paper-IV : Special Relativity-II

P. Pages : 2 **GUG/W/24/13362**

Time : Three Hours * 5 0 8 2 * Max. Marks : 60

Notes : 1. Solve all the questions.
2. Each questions carry equal marks.

UNIT – I

1. a) If $f = a_{rs}x^r x^s$ then show that $\frac{\partial f}{\partial x^r} = (a_{rs} + a_{sr})x^s$ and $\frac{\partial^2 f}{\partial x^r \partial x^s} = a_{rs} + a_{sr}$. **6**

b) Show that if A^m, B_{nrs} are tensors then $A^m B_{mrs}$ is also tensor. **6**

OR

c) For a mixed tensor T^m_{nrs} of order 4 show that T^n_{nrs} is a tensor of order 2. **6**

d) Show that δ^r_s is a mixed tensor of order two let A^{pq}_{rst} be a tensor. Choosing p = t, q = s show that A^{pq}_{rqp} is also a tensor. What is its rank? **6**

UNIT – II

2. a) Show that $\Gamma^m_{mn} = \left(\log\sqrt{g}\right)_n$ **6**

b) For a scalar A, then prove that $\frac{\delta A}{\delta u} = \frac{dA}{du}$. **6**

OR

c) Find nonvanishing components of Christoffel symbols of second kind for $ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$. **6**

d) Show that $\frac{\delta T^r}{\delta u}$ is a contravariant vector. **6**

UNIT – III

3. a) Derive the expression for force in the transverse and longitudinal mass. **6**

b) Show that $p^2 - E^2/c^2$ is invariant whose numerical value is $-m_0^2c^2$. **6**

OR

c) Obtain the equation of motion of a free particle. **6**

d) Find the expression for four velocity in component form. **6**

UNIT – IV

4. a) Obtain the Maxwell's equations of electromagnetic theory in vacuum in the component form. **6**

b) Show that the Hamiltonian for a charged particle moving in an electromagnetic field is **6**

$$H = \left[m_0^2c^4 + c^2\left(p - \frac{e}{c}A \right)^2 \right]^{\frac{1}{2}} + e\phi$$

OR

c) An electromagnetic field is purely magnetic in an inertial frames then describe the field in inertial frames. **6**

d) Show that **6**

i) $Ey' = \alpha\left(Ey - \frac{\nu}{c}Hz \right)$ and ii) $Ez' = \alpha\left(Ez + \frac{\nu}{c}Hy \right)$

5. Solve **any six.**

a) Show that $a_{mn}\, x^m x^n = 0$ for a skew symmetric tensor a_{mn}. **2**

b) Define the contraction of tensor. **2**

c) Show that $[mn, r] = [nm, r]$. **2**

d) Define Einstein tensor. **2**

e) Prove that $g_{ij}\, u^i\, u^j = 1$. **2**

f) Define the four force. **2**

g) Write the equation $E = -\text{grad}\,\phi - \frac{1}{c}\frac{\partial A}{\partial t}$ in component form. **2**

h) Show that $E' = E$. **2**

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