M.Sc. - II (Mathematics) (New CBCS Pattern) Semester-IV

PSCMTH17 : Partial Differential Equations

P. Pages : 2

Time : Three Hours

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GUG/W/24/13768

Max. Marks : 100

Notes : 1. Solve all the **five** questions.
2. All questions carry equal marks.

UNIT - I

1. a) Find the complete integral of the partial differential equation $f \equiv x^2p^2 + y^2q^2 - 4 = 0$. **10**

b) Prove that necessary and sufficient condition for the integrability of **10**

$dz = \phi(x, y, z) + \varphi(x, y, z)$ is $[f, g] = \frac{\partial(f, g)}{\partial(x, p)} + p\frac{\partial(f, g)}{\partial(z, p)} + \frac{\partial(f, g)}{\partial(y, q)} + q\frac{\partial(f, g)}{\partial(z, q)} = 0$.

OR

c) Find the General Solution of $x\left(y^2 - z^2\right)p - y\left(z^2 + x^2\right)q = \left(x^2 + y^2\right)z$. **10**

d) Solve by using Jacob's method $z^2 + zu_z - u_{x^2} - u_{y^2} = 0$. **10**

UNIT - II

2. a) Find the integral surface of the partial differential equation $(x - y)p + (y - x - z)q = z$ through the circle $z = 1, x^2 + y^2 = 1$. **10**

b) Find by the method of characteristics the integral surface of pq = xy which passes through line z = x, y = 0. **10**

OR

c) Find the solution of the equation. **10**

$z = \frac{1}{2}\left(p^2 + q^2\right) + (p - x)(q - y)$ which passing through x-axis.

d) Solve the IVP for the quasilinear equation $zz_x + z_y = 1$ contains the initial curve **10**

$c : x_0 = s, y_0 - s, z_0 = \frac{s}{2}, 0 \le s \le 1$.

UNIT - III

3. a) Reduce the equation $y^2u_{xx} - 2x_y\,u_{xy} + x^2u_{yy} = \frac{y^2}{x}u_x + \frac{x^2}{y}u_y$ into canonical form and solve it. **10**

b) Derive the solution of the wave equation or solution of vibration of string of finite length. **10**

OR

c) Prove that for the equation $L_u \equiv u_{xy} + \frac{1}{4}u = 0$. The Riemann function is $\nu(x, y, \alpha, \beta) = J_0\left(\sqrt{(x-\alpha)(y-\beta)}\right)$ where J_0 denotes the Bessel's function of the first kind of order zero. **10**

d) Reduce the equation $(n-1)^2u_{xx} - y^{2n}\,u_{yy} = n_y{}^{2n-1}\,u_y$, where n is an integer to a canonical form. **10**

UNIT - IV

4. a) State and prove that Duhamel's principle. **10**

b) Solve **10**

$u_t = u_{xx}\ 0 < \ < 1, t > 0$

$u(0,t) = u(1,t) = 0$

$u(0) = x(1-x), 0 \le \ \le 1$

OR

c) Show that the solution for the Dirichlet problem for a circle of radius is given by the Poisson integral formula $U(\rho,\theta) = \frac{1}{2\pi}\int_0^{2\pi}\frac{1-\rho^2}{1-2\rho\mathrm{Cos}(\theta-\tau)+\rho^2}f(\tau)dc$, where $\rho = \frac{r}{a}$. **10**

d) State and prove the maximum and minimum principles. **10**

5. a) Find the complete integral of $f(p,q) \equiv p + q - pq = 0$. **5**

b) Solve the Cauchy problem for $2z_x + yz_y = z$ for the initial data curve $c : x_0 = s, y_0 = s^2, z_0 = s, 1 \le s \le 2$. **5**

c) Derive a wave equation in canonical form. **5**

d) State The Dirichlet problem and Neumann problem. **5**

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