B.Sc. - I (CBCS Pattern) Semester-I

USMT-01 - Mathematics Paper-I - Differential and Integral Calculus

P. Pages : 2 **GUG/W/24/11556**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all the five questions.
2. Each equation carries equal marks.

UNIT – I

1. a) Prove that if $\lim_{x \to x_0} f(x)$ exists then it is unique. **6**

b) $\text{Let } f(x) = \begin{cases} x.\sin\frac{1}{x} & , x \neq 0 \\ 2 & , x = 0 \end{cases}$ **6**

Show that f(x) has removable discontinuity at x = 0.

OR

c) If f(x) is differentiable at $x = x_0$ then prove that it is continuous at $x = x_0$. **6**

d) If $y = (x^2 - 1)^n$ then prove that $(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$. **6**

UNIT – II

2. a) Verify the Rolle's theorem for the function $f(x) = x^2 + x - 6 \text{ in } [-3, 2]$. **6**

b) If f and g are real functions on [a, b] which are differentiable in (a, b) then prove that there is a point $c \in (a, b)$ such that **6**

$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}$$

Where $g(b) \neq g(a)$ and $f'(x), g'(x)$ are not simultaneously zero.

OR

c) Show that $\log(x + h) = \log h + \frac{x}{h} - \frac{x^2}{2h^2} + \frac{x^3}{3h^3} - \text{........}$ **6**

d) Obtain Maclaurin's series for f(x) = sinx. **6**

UNIT – III

3. a) Prove that $\int_0^\infty x^{n-1} e^{-kx} dx = \frac{n}{k^n}$, where n, k > 0 are constants. **6**

b) Prove that $\beta(m,n)=\int_0^{\infty}\frac{x^{m-1}}{(1+x)^{m+n}}\,dx$. **6**

OR

c) Prove that $\lim\limits_{x\to 0}\frac{(1+x)^{1/x}-e}{x}=-\frac{e}{2}$. **6**

d) Prove that $\lim\limits_{x\to 0}\log_{\tan x}\tan 2x=1$. **6**

UNIT – IV

4. a) Let f(x,y) and g(x,y) are continuous on region D of area A. Then prove that **6**

i) $\iint_D cf(x,y)dA=c\iint_D f(x,y)dA$, c is a constant.

ii) $\iint_D [f(x,y)+g(x,y)]dA=\iint_D f(x,y)dA+\iint_D g(x,y)dA$

b) Evaluate $\int_{-2}^{2}dx\int_{y^2-1}^{3}(x+2y)dy$. **6**

OR

c) Evaluate $\iint_D x^2\,dxdy$, where D is the region in the first quadrant bounded the by the hyperbola xy = 16 and the line y = x, y = 0 and x = 8. **6**

d) Evaluate $\iint xy(x+y)\,dxdy$ over the area between $y=x^2, y=x$. **6**

5. Solve **any six.**

a) Write $\in-\delta$ definition of limit of a function. **2**

b) If y = Asinmx + Bcosmx then prove that $y_2+m^2y=0$. **2**

c) State the Lagrange's mean value theorem. **2**

d) Verify Rolle's theorem for the function $f(x)=x^2$ in [-1, 1]. **2**

e) Prove that $\Gamma_1=1$. **2**

f) Evaluate B (5, 3). **2**

g) Prove that $\iint_D cf(x,y)\,dA=c\int_D f(x,y)\,dA$, c is a constant. **2**

h) Evaluate $\int_0^1\int_1^3 xy^2\,dydx$. **2**

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