M.Sc. (Physics) (CBCS Pattern) Semester-I

PSCPHYT02 - Complex Analysis and Numerical Methods

P. Pages : 4 **GUG/W/24/11180**

Time : Three Hours * 4 2 7 2 * Max. Marks : 80

Note : All questions are compulsory.

Either:

1. a) If $z = \cos\theta + i\sin\theta$, prove that, **8**

i) $\dfrac{2}{1+z} = 1 - i\tan\dfrac{\theta}{2}$

ii) $\dfrac{1+z}{1-z} = i\cot\dfrac{\theta}{2}$

b) Determine whether the following functions are analytic or not? **8**

i) $x^2 + iy^2$

ii) $2xy + i(x^2 - y^2)$

iii) $\dfrac{x - iy}{x^2 + y^2}$

iv) $xy + iy^2$

OR

e) Find the integral: **8**

i) $\int_C \dfrac{3z^2 + 7z + 1}{z+1}\,dz$, where, C is the circle $|z| = 1/2$.

ii) $\oint_C \dfrac{2z^2 + 5}{(z+2)^3(z^2+4)}\,dz$, where C is the square with vertices at $1+i, 2+i, 2+2i, 1+2i$.

f) **8**

If $f(z_1) = \int_C \dfrac{3z^2 + 7z + 1}{z - z_1}\,dz$, where C is the circle $x^2 + y^2 = 4$, find the values of (i) $f(3)$ and (ii) $f(1-i)$.

Either:

2. a) Determine the residue of the following: **8**

i) $f(z) = \dfrac{z^3}{(z-1)^4(z-2)(z-3)}$ at its simple pole and

ii) $f(z) = \dfrac{z^2}{(z-1)(z-2)(z-3)}$ at $z = 1, 2, 3$.

b) Explain method of finding residue at **8**

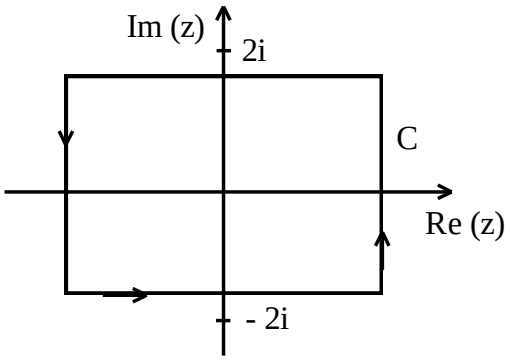
i) Simple pole and

ii) Pole of order n.

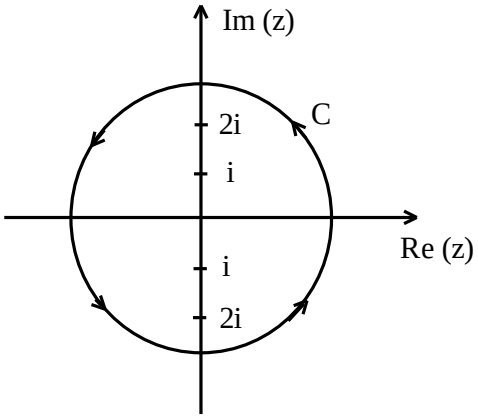
OR

e) Evaluate the following, Integral using residue theorem: $\int_C \dfrac{12z-7}{(z-1)^2(2z+3)}\,dz$, where C is the circle (i) $|z| = 2$ and (ii) $|z+i| = \sqrt{3}$. **8**

f) Compute (i) $\int_C \dfrac{\cos(z)}{z(z^2+\theta)}dz$ over the counter shown **8**



and (ii) $\int \dfrac{z}{z^2+4}dz$ over the curve C shown below:



Either:

3. a) Find the root correct to three significant figures of the equation $x^3 - x - 4 = 0$ using the method of false position. **8**

b) Find the root of the equation $x^3 - 2x^3 + 3x - 5 = 0$ using Newton's Raphson method corrected upto three decimal places. **8**

OR

e) Find the real root of the equation $x^3 + x^2 - 1 = 0$ on the interval (0, 1) corrected upto three decimal places using Iteration Method. **8**

f) In the following table, the values of y is represented for a value of x. Find the first and tenth term using Newton's forward and backward difference formula. **8**

x	3	4	5	6	7	8	9
y	4.8	8.4	14.5	23.6	36.2	52.8	73.9

Either:

4. a) Evaluate $\int_1^2 e^{-1/2x}\, dx$ divided into four subintervals by applying Simpson's $\frac{1}{3}^{rd}$ and $\frac{3}{8}^{th}$ rule. **8**

b) Certain corresponding values of x and $\log_{10}{}^x$ are : **8**

x	300	304	305	307
$\log_{10}{}^x$	2.4771	2.4829	2.4843	2.4871

Find $\log_{10}{}^{301}$ using Lagrange's interpolation method.

OR

e) Given $\frac{dy}{dx} = xy$ with $y(1) = 5$ find the solution in the interval [1, 1.5] for step size h = 0.1 using Euler's method. **8**

f) Find y(1) using Runge-Kutta method for order two by solving the equation $\frac{dy}{dx} = -2xy^2$, $y(0)$ with step size 0.1. **8**

5. Attempt all of the following.

a) Describe geometrical representation of sum and difference of two complex numbers. **4**

b) Prove that the function $f(z) = \sum_{n \in Z} \frac{1}{(n+z)^2}$ is meromorphic on C. **4**

c) From the following table, estimate the number of students who obtain marks 45. Using Newtons forward Interpolation formula. **4**

x	40	50	60	70	80
y	31	73	124	159	190

d) Determine arc of the curve using following table of values from $x = 7.47$ to $x = 7.52$ through trapezoidal rule. **4**

x	7.47	7.48	7.49	7.50	7.51	7.52
f(x)	1.93	1.95	1.98	2.01	2.03	2.06

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