B.Sc.- II (CBCS Pattern) Semester-IV

USMT-07 - Mathematics Paper-I - Algebra

P. Pages : 2 **GUG/W/24/12014**

Time : Three Hours * 4 7 5 3 * Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT - I

1. a) If G is a group in which $(a,b)^i = a^i b^i$, for three consecutive integer $i\ \forall a,b \in G$, show that G is abelian. **6**

b) Let G be a group of all real 2 x 2 matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with $ad - bc \neq 0$ under matrix multiplication. Let $H = \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} \in G \mid ad \neq 0 \right\}$. Show that H is a subgroup of G. **6**

OR

c) Let θ be a permutation of a set S. Define a relation $\sim$ on S as follow $a, b \in S,\ a \sim b \Leftrightarrow \theta^i a = b$, for some integer i. Then prove that this relation is an equivalence relation. **6**

d) Find the orbits and cycles of the following permutation. **6**
$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 6 & 4 \end{pmatrix}$.

UNIT - II

2. a) Show that the set of inverses of the elements of a right coset is a left coset. **6**

b) Prove that any two right coset of a subgroup are either disjoint or identical. **6**

OR

c) Prove that N is a normal subgroup of $G \Leftrightarrow NaNb = Nab, \quad \forall a, b \in G$. **6**

d) If H is a subgroup of G and N is a normal subgroup of G. Then show that $H \cap N$ is a normal subgroup of H. **6**

UNIT - III

3. a) Let G be any group, g a fixed element in G. Define $\phi : G \to G$ by $\phi(x) = gxg^{-1}$. Prove that ϕ is an isomorphism of G onto G. **6**

b) If ϕ is a homomorphism of G into G' with kernel K. Then prove that K is a normal subgroup of G. **6**

OR

c) Prove that any infinite cyclic group is isomorphic to the additive group of integer. **6**

d) If M, N are normal subgroups of G. Prove that $\frac{NM}{M} \approx \frac{N}{N \cap M}$. **6**

UNIT - IV

4. a) Prove that a ring R is commutative $\text{iff}'(\dot{a}+b)^2 = a^2 + 2ab + b^2$. **6**

b) Show that the set of matrices $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ is a subring of the ring R of 2×2 matrices with integral elements. Here a, b, c are integers. **6**

OR

c) Show that the set $A = \left\{a + b\sqrt{3} \mid a, b \in Q\right\}$ is a field under the addition and multiplication of rational numbers. **6**

d) If R is a ring in which $x^2 = x, \ \forall x \in R$, then prove that R is a commutative ring of characteristic 2. **6**

5. Solve **any six.**

a) Define a group. **2**

b) If G is group, then prove that for every $a \in G, \left(a^{-1}\right)^{-1} = a$. **2**

c) State the Lagrange's theorem. **2**

d) Show that every subgroup of an abelian group is normal. **2**

e) Define Kernel of a homomorphism. **2**

f) If ϕ is a homomorphism of a group G into G'. Then prove that $\phi(e) = e'$. **2**

g) If R is a ring with unity, then show that this unity is unique. **2**

h) Define an integral domain. **2**

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