M.Sc. Second Year (Mathematics) (New CBCS Pattern) Semester-IV

PSCMTH18 : Integral Equations

P. Pages : 2 **GUG/W/24/13769**

Time : Three Hours Max. Marks : 100

UNIT – I

1. a) Show that the function $u(x) = (1+x^2)^{-3/2}$ is a solution of the Volterra equation **10**

$$u(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{(1+x^2)} u(t)\,dt$$

b) Form an integral equation corresponding to the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$ with initial conditions $y(0) = 0,\ y'(0) = -1$. **10**

OR

c) Form an integral equation corresponding to the differential equation given by $\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 3y = 0$ with initial conditions $y(0) = 1,\ y'(0) = 0$. **10**

d) Show that $u(x) = \cos 2x$ is a solution of the integral equation **10**

$$u(x) = \cos x + 3\int_0^\pi k(x,t)\,u(t)\,dt \quad \text{where } k(x,t) = \begin{cases} \sin x \cos t & ; \quad 0 \le x \le t \\ \cos x \sin t & ; \quad t \le x \le \pi \end{cases}$$

UNIT – II

2. a) If k(x, t) is symmetric and $f_0(x)$ and $f_1(x)$ are eigen functions of k(x, t) corresponding to eigen values λ_0 and λ_1 respectively $[\lambda_0 \ne \lambda_1]$. Then $f_0(x)$ and $f_1(x)$ are orthogonal on (a, b) i.e. $\int_a^b f_0(x) f_1(x)\,dx = 0$ **10**

b) Find the eigen values and the corresponding eigen functions of the Homogeneous integral equation $u(x) = \lambda \int_0^1 \sin \pi x \cos \pi t\, u(t)\,dt$ **10**

OR

c) Solve the following integral equation $u(x) = x + \lambda \int_0^1 (1+x+t)\,u(t)\,dt$. **10**

d) Solve $u(x) = 1 + \int_0^1 (1+e^{x+t})\,u(t)\,dt$ **10**

UNIT – III

3. a) State and prove Schwarz Inequality. **10**

b) State and prove Bessel's Inequality. **10**

OR

c) Solve the following Homogeneous Fredholm integral equation using schimdt solution. **10**

$$f(x) = \lambda \int_0^1 e^x e^t f(t)\,dt$$

d) Solve the integral equation $u(x) = e^x + \lambda \int_0^1 (5x^2 - 3)t^2 . u(t)\,dt$ **10**

UNIT – IV

4. a) Solve $u(x) = f(x) + ½ \int_0^1 e^{x-t} u(t)\,dt$ **10**

b) Solve $u(x) = 1 + \lambda \int_0^{\pi} \sin(x+t) u(t)\,dt$ **10**

OR

c) Solve the following Fredholm integral equation of the second kind **10**

$u(x) = 2x + \lambda \int_0^1 (x+t) u(t)\,dt$ by the method of successive approximation by taking $u_0(x) = 1$.

d) Solve the following integral equation $u(x) = 1 + \lambda \int_0^1 (x+t) u(t)\,dt$ by the method of successive approximation upto third order. **10**

5. a) Definition of Fredholm Integral Equation of the first kind and second kind. **5**

b) Define orthogonal functions and separable Kernel. **5**

c) If k(x, t) is real and symmetric, continuous and identically not equal to zero, then prove that all the characteristics constants are real. **5**

d) Find the solution of the integral equation $u(x) = \sin x + 2 \int_0^x e^{x-t} u(t)\,dt$ with the help of resolvent kernel. **5**

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