USMT-02 - Mathematics Paper-II - Differential Calculus and Trigonometry

P. Pages : 2 **GUG/W/24/11557**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all the **five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) Using $\in - \delta$ definition of limit of a function, prove that- **6**

$$\lim_{(x,y)\to(4,-1)} (3x-2y) = 14$$

b) Prove that the function $f(x,y) = x + y$ is continuous, $\forall (x,y) \in R^2$. **6**

OR

c) If $u = \dfrac{e^{x+y+z}}{e^x + e^y + e^z}$ then show that $u_x + u_y + u_z = 2u$. **6**

d) If $u = F(x-y, y-z, z-x)$ then prove that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = 0$. **6**

UNIT – II

2. a) If $u = \log\left(\dfrac{x^4 - y^4}{x - y}\right)$, prove that : **6**

i) $xu_x + yu_y = 3$ ii) $x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} = -3$.

b) If $xu = yz, yv = xz$ and $zw = xy$ then find $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}$. **6**

OR

c) Obtain the expansion of $f(x,y) = x^2 - y^2 + 3xy$ at the point (1,2). **6**

d) Explain the Hungarian method for the solution of the assignment problems. **6**

UNIT – III

3. a) Find the tangent at the origin to the curve $x^2 y^2 = a^2 (x^2 - y^2)$. **6**

b) Find the radius of curvature at the point (x,y) on the curve $y^2 = 4ax$. **6**

OR

c) Find the asymptotes of the curve: **6**

$3x^3+2x^2y-7xy^2+2y^3-14xy+7y^2+4x+5y=0$.

d) Trace the curves : **6**

i) $y^2(a-x)=x^3$ ii) $x^2y^2=a^2\left(y^2-x^2\right)$.

UNIT – IV

4. a) Prove that $(1+i)^n+(1-i)^n=2^{\left(\frac{n}{2}\right)+1}\cos\left(\frac{n\pi}{4}\right)$. **6**

b) If α and β are the roots of the equation $x^2-2x+4=0$, prove that **6**

$\alpha^n+\beta^n=2^{n+1}\cos\frac{n\pi}{3}$.

OR

c) Find all the values of $32^{1/5}$. **6**

d) If $\sin(\alpha+i\beta)=x+iy$ then $\frac{x^2}{\cosh^2\beta}+\frac{y^2}{\sinh^2\beta}=1$ and $\frac{x^2}{\sin^2\alpha}-\frac{y^2}{\cos^2\alpha}=1$. **6**

5. a) Using algebra of limits, prove that $\lim_{(x,y)\to(1,2)}\left(x^2+xy-2x-y\right)=-1$. **2**

b) If $z=f\left(x^2-y^2\right)$, show that $yz_x+xz_y=0$. **2**

c) State Euler's theorem for homogeneous function f(x,y). **2**

d) Define stationary point of f(x,y). **2**

e) Find the tangent and normal at (1,3) to the curve $y=x^3+2$. **2**

f) Find ρ at the point (s,Ψ) of the curve $s=4a\sin\Psi$. **2**

g) Express – 1 – i in polar form. **2**

h) Prove that cosiz = icosh z. **2**

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