M.Sc. Second Year (Mathematics) (New CBCS Pattern) Semester-III

PSCMTH12 - Functional Analysis

P. Pages : 2 **GUG/W/24/13756**

Time : Three Hours * 5 4 3 3 * Max. Marks : 100

Notes : 1. All **five** questions are compulsory.
2. Each question carries equal marks.

UNIT – I

1. a) State and prove The Hahn – Banach Theorem. **10**

b) If N is Normed linear space, then prove that The closed unit sphere S* in N* is a compact Hausdorff space in the weak* topology. **10**

OR

c) Let M be a linear subspace of a Normal linear space N, and let f be a functional defined on M. If x_0 is a vector not in M_1 and If $M_0 = M + [x_0]$ is the linear subspace spanned by M and x_0 then prove that f can be extended to a functional F_0 defined on M_0 such that $\| f_0 \| = \| f \|$. **10**

d) If M is a closed linear subspace of Normed linear space N and x_0 is a vector Not in M. Then there exists a functional f_0 in N* such that $f_0(m) = 0$ and $f_0(x_0) \neq 0$. **10**

UNIT – II

2. a) Prove that A closed convex subset C of a Hilbert space H contains a unique vector of smallest Norm. **10**

b) If x and y are any two vectors in a Hilbert space, Then prove that $|(x, y)| \leq \| x \| \cdot \| y \|$ **10**

OR

c) State and prove the uniform Boundedness Theorem. **10**

d) If B and B' are Banach spaces, and If T is a linear transformation of B into B', Then prove that T is continuous $\Leftrightarrow$ its graph is closed. **10**

UNIT – III

3. a) Let H be a Hilbert space, and let f be an arbitrary functional in H*. Then prove that there exist a unique vector y in H such that **10**

$f(x) = (x,y)$

for every x in H

b) If T is an operator on H, Then prove that T is Normal If and only if its Real and imaginary parts commute. **10**

OR

c) Prove that An operator T on H is self-adjoint $\Leftrightarrow (T_x, x)$ is real for all x **10**

d) If N_1 and N_2 are Normal operators on H with The property that either commutes with the adjoint of the other, Then prove that N_1 & N_2 and N_1, N_2 are Normal. **10**

UNIT – IV

4. a) Prove that let B be a Basis for H, and T an operator who be matrix relative to B is $[\alpha ij]$. Then T is non-singular $\Leftrightarrow [\alpha ij]$ is Non-singular, and in this case $[\alpha ij]^{-1} = \left[T^{-1}\right]$ **10**

b) If T is Normal, Then prove that the Mi's span H. **10**

OR

c) If T is Normal, then prove that x is an eigenvector of T with eigenvalue $\lambda \Leftrightarrow x$ is an eigen vector of T* with eigenvalue $\bar{\lambda}$. **10**

d) If $B = \{\ell_i\}$ is a Basis for H. Then prove that the Mapping $T \rightarrow [T]$, which assigns to each operator T its matrix relative to B, its an isomorphism of The Algebra (BCH) outo the total matrix Algebra An. **10**

5. a) Define the natural imbedding of N in N** **5**

b) Definition of orthogonal complements and ortho Normal bets. **5**

c) If A_1 and A_2 are self adjoint operators on H, then prove that their product A_1, A_2 is self - adjoint E> $A_1 \, A_2 = A_2 \, A_1$ **5**

d) Explain matrix representation of a linear operator on a vector space. **5**

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