

M.Sc. - II (Mathematics) (New CBCS Pattern) Semester-IV
PSCMTH19B - Optional Paper : Cosmology

P. Pages : 2

Time : Three Hours



GUG/W/24/13771

Max. Marks : 100

- Notes : 1. Solve **all five** questions.
2. All questions carry equal marks.

UNIT-I

1. a) Explain Doppler shift in de sitter universe. 10
b) Explain the geometry of the Einstein universe. 10

OR

- c) Explain the geometry of the de sitter universe. 10
d) Explain pressure & density of de sitter universe. 10

UNIT-II

2. a) Derive Robertson Walker line element. 10
b) Prove that the quantity $\frac{1}{a^2}$ in 10

$$ds^2 = -e^{g(t)} \left[\frac{dr^2}{1 - \frac{r^2}{a^2}} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right] + dt^2$$

is the Reimannian curvature of the spatial extent at any given time t.

OR

- c) Prove that the spatial extent of 10

$$ds^2 = -e^{g(t)} \left[\frac{dr^2}{1 - \frac{r^2}{a^2}} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] + dt^2$$

at any given time is the whole three dimensional spherical surface

$$(x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2 = a^2 \text{ embedded in the four dimensional Euclidean space } (x^1, x^2, x^3, x^4).$$

- d) Explain Radial motion of a particle in Robertson walker model. 10

UNIT-III

3. a) Explain fundamental equations of dynamical cosmology. **10**
- b) Prove that the matter dominated era of the universe is governed by the equation **10**
- $$\left(\frac{R}{R_0}\right)^2 = H_0^2 \left[1 - 2q_0 + 2q_0 \frac{R_0}{R}\right]$$

Where the symbols have their conventional meanings.

OR

- c) Show that the closed isotropic model of the universe filled with dust can be put in the form **10**
- $$ds^2 = a^2(1 - \cos \tau)^2 \left[-d\alpha^2 - \sin^2 \alpha (d\theta^2 + \sin^2 \theta d\phi^2) + d\tau^2 \right]$$

Where τ is given in terms of cosmic time t by $t = a(\tau - \sin \tau)$.

- d) Discuss an open isotropic model of the universe given by the RW metric **10**
- $$ds^2 = R^2(t) \left[-d\alpha^2 - \sinh^2 \alpha (d\theta^2 + \sin^2 \theta d\phi^2) \right] + dt^2$$
- with reference to dust distribution.

UNIT-IV

4. a) Explain apparent luminosity & luminosity distance. **10**
- b) Explain angular diameter & angular diameter distance. **10**

OR

- c) Explain proper motion & proper motion distance. **10**
- d) Explain Parallax & parallax distance. **10**
5. a) Compare Einstein & de sitter models. **5**
- b) Explain the visible universe is isotropic. **5**
- c) Obtain the relation between k , ρ , & H_0 . **5**
- d) Explain light paths. **5**
