B.Sc. (CBCS Pattern) Semester-IV

USMT-08 - Mathematics Paper-II - Elementary Number Theory

P. Pages : 2 **GUG/W/24/12015(S)**

Time : Three Hours Max. Marks : 60

Notes :
1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) Find all positive integer n (<17) for which $n!+(n+1)!+(n+2)!$ is an integral multiple of 49. **6**

b) If x and y are odd, prove that x^2+y^2 is not a perfect square. **6**

OR

c) Find the g.c.d. of 275 and 200 and express it in the form 275x+200y. **6**

d) For positive integers a and b prove that $(a,b)[a,b]=ab$. **6**

UNIT – II

2. a) Prove that, there are an infinite number of primes. **6**

b) Prove that, every positive integer greater than one has at least one prime divisor. **6**

OR

c) Prove that $\left(a^2,b^2\right)=c^2$ if (a, b) = c, c > 0. **6**

d) Prove that, for any positive integer n there are at least n consecutive composite integers. **6**

UNIT – III

3. a) Let a, b, c be integers such that $a\equiv b \pmod m$ then prove that **6**

i) $(a+c)\equiv(b+c)\pmod m$

ii) $ac\equiv bc \pmod m$

b) If $a\equiv b\pmod m$ then prove that $a^n\equiv b^n \pmod m$, $\forall n\in N$. **6**

OR

c) Let f denote a polynomial with integral coefficient if $a \equiv b \pmod{m}$ then prove that $f(a) \equiv f(b) \pmod{m}$. **6**

d) Solve the system of three congruence $x \equiv 2 \pmod{3}$, $x \equiv 3 \pmod{5}$, $x \equiv 2 \pmod{7}$. **6**

UNIT - IV

4. a) Let P be a prime and K is a positive integer Then prove that- $\phi\left(p^k\right) = p^k - p^{k-1}$. **6**

b) Find all the positive integers x and y such that $x^{\phi(y)} = y$. **6**

OR

c) Prove that the Mobius μ - function is multiplicative. **6**

d) If x, y, z is a primitive Pythagorean triple then prove that- $(x, y) = (y, z) = (z, x) = 1$. **6**

5. Solve **any six.**

a) Let a, b and c be integers such that a/b and b/c then prove that a/c. **2**

b) Let a and b be any two integers that are not both zero then prove that their g.c.d. is unique. **2**

c) Define prime integer and composite integer. **2**

d) If p is a prime and p/ab then prove that p/a or p/b. **2**

e) Show that if a, b, m and n are integers such that m>0, n>0, m/n and $a \equiv b \pmod{n}$ then prove that $a \equiv b \pmod{m}$. **2**

f) Let $a_1, b_1 \in z$ such that $a_1 \equiv b_1 \pmod{m}$, then prove that $ca_1 \equiv cb_1 \pmod{m}, c \in z$. **2**

g) If P is prime then prove that $\phi(p) = p - 1$. **2**

h) If a, b, c is a primitive Pythagorean triple then prove that ka, kb, kc, $k \in z$ is a Pythagorean triple, $\forall k \in z$. **2**

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