B.Sc. - III (CBCS Pattern) Semester-VI

021B - (DSE-VII) Mathematics-III : Linear Programming and Transportation Problems

P. Pages : 4 **GUG/W/24/13364**

Time : Three Hours * 5 0 8 4 * Max. Marks : 60

Notes : 1. Solve all the **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that the intersection of any two convex sets is also a convex set. **6**

b) Write the following Linear program in its standard form. **6**

Minimize : $Z = 25x_1 + 30x_2$

Subject to : $4x_1 + 7x_2 \geq 1$

$8x_1 + 5x_2 \geq 3$

$6x_1 + 9x_2 \geq 2$

with x_1, x_2 unrestricted in sign.

OR

c) Solve the following LPP by graphical method. **6**

Max $Z = 4x_1 + 3x_2$

Subject to $2x_1 + x_2 \leq 1000$

$x_1 + x_2 \leq 800$

$x_1 \leq 400$

$x_2 \leq 700$

with $x_1, x_2 \geq 0$.

d) Determine whether the set $\left\{ [1,1,3,1]^T, [1,2,1,1]^T, [1,0,0,1]^T \right\}$ is linearly independent. **6**

UNIT – II

2. a) Solve the following LPP problem by simplex method. **6**

Maximize : $Z = x_1 + x_2$

Subject to : $x_1 + 5x_2 \leq 5$

$2x_1 + x_2 \leq 4$

with $x_1, x_2 \geq 0$.

b) Solve the following LPP by Big-M method. **6**

Maximize : $Z = 6x_1 + 4x_2$

Subject to : $2x_1 + 3x_2 \leq 30$

$3x_1 + 2x_2 \leq 24$

$x_1 + x_2 \geq 3$

with $x_1, x_2 \geq 0$.

OR

c) Solve the following LPP by two-phase method **6**

Maximize : $Z = 5x_1 + 3x_2$

Subject to : $2x_1 + x_2 \leq 1$

$x_1 + 4x_2 \geq 6$

with $x_1, x_2 \geq 0$.

d) Prove that the dual of the dual is the primal. **6**

UNIT – III

3. a) Prove that a necessary and sufficient condition for the existence of a feasible solution to the transportation problem is $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. **6**

b) Obtain an initial feasible solution to the following transportation problem using north-west corner rule. **6**

		D	E	F	G	Available
	A	11	13	17	14	250
Origin	B	16	18	14	10	300
	C	21	24	13	10	400
Requirement		200	225	275	250	

OR

c) Determine an initial basic feasible solution to the transportation problem using Vogel's approximation method. **6**

	A	B	C	D	Supply
I	1	2	1	4	30
II	3	3	2	1	50
III	4	2	5	9	20
Demand	20	40	30	10	

d) Find the initial basic solution of the following transportation problem by matrix minima method. **6**

Warehouse factory	w_1	w_2	w_3	w_4	Factory capacity
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Warehouse requirement	5	8	7	14	34

UNIT – IV

4. a) Prove that in an assignment problem if we add (or subtract) a constant to every element of any row (or column) of the cost matrix [Cij] then an assignment that minimizes the total cost on one matrix will also minimizes the total cost on the other matrix. **6**

b) HMT Ltd. decides to make four subassemblies through four contractors. Each contractor is to receive only one subassembly. The cost of each subassembly is determined by the bids submitted by each contractor and is shown in the following table in hundreds of rupees. Assign the different subassemblies to contractors to minimize the total cost. **6**

		Contractors			
		1	2	3	4
Sub assemblies	1	15	13	14	17
	2	11	12	15	13
	3	13	12	10	11
	4	15	17	14	16

OR

c) Maximize $Z = x(5\pi - x)$ on $[0, 20]$. **6**

d) Define concave function. Show that $f(x) = x^3 - 6x^2 + 9x + 6$ is strictly concave on $(-\infty, 2)$ and strictly convex on $(2, \infty)$. **6**

5. Solve **any six.**

a) Write the general form of linear programming problem. **2**

b) Define slack variables. **2**

c) What is the condition of optimality in simplex method? **2**

d) Write dual of the programme **2**

Maximize : $Z = 5x_1 + 3x_2$

Subject to : $3x_1 + 5x_2 \leq 15,$

$5x_1 + 2x_2 \leq 10$

with $x_1, x_2 \geq 0.$

e) Define feasible solution to a transportation problem. **2**

f) Write mathematical formulation of transportation problem. **2**

g) Define global maximum. **2**

h) What is balanced assignment problem? **2**

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