M.Sc.(Physics) (NEP Pattern) Semester-I

NEP-236-1 / 01MSCPH4.1 - DSE Paper-IV
Complex Analysis and Numerical Methods

P. Pages : 3 **GUG/W/24/15137**

Time : Three Hours * 6 0 2 0 * Max. Marks : 80

Either:

1. a) Determine whether the following equations are analytic or not? **8**

i) $\frac{1}{z}$

ii) $e^{x}(\cos y + i\sin y)$

b) Prove the necessary conditions for a function $f(z) = u + iv$ to be analytic at all the points in a region R are: **8**

i) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and

ii) $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ provided $\frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}$ exist

OR

e) If $f(z)$ is analytic within and an a closed curve c, and if a is any point within c, then prove that, **8**

$f(a) = \frac{1}{2\pi i}\int_{z} \frac{f(z)}{z-a}\,dz$.

f) Using Cauchy integral formula evaluate $\oint_{c} \frac{z^2+1}{z^2-1}\,dz$, where c is circle. **8**

For i) $|z| = 3/2$

ii) $|z-1| = 1$

Either:

2. a) Define the term 'singularity' of a function? Find the singularity of the following functions. **8**

i) $f(z) = \sin\frac{1}{z}$ and

ii) $f(z) = \frac{e^{1/z}}{z^2}$

b) Find the residue of the following functions: **8**

i) $f(z)=\dfrac{z^2}{(z+1)^2(z-2)}$ at its double pole.

ii) $f(z)=\dfrac{1}{\left(z^2+1\right)^3}$ at $z=i$.

OR

e) Evaluate the following integral using residue theorem: **8**

i) $\int_c \dfrac{1+z}{z(2-z)}\,dz$, where, c is the circle $|z|=1$.

ii) $\int_c \dfrac{4-3z}{z(z-1)(z-2)}\,dz$, where c is the circle $|z|=\dfrac{3}{2}$.

f) Defining term 'Meromorphic function' prove that **8**

$$\frac{1}{2\pi i}\int_c \frac{f'(z)}{f(z)}dz = N-P$$

Where, N is the number of zeros and P is the number of poles inside c

Either:

3. a) Find the root of the equation $x^3-18=0$ using Bisection method corrected upto three decimal points. **8**

b) Start with $x_1=4$ and $x_2=2$ and use the secant method to find the root of the polynomial function $f(x)=x^2-4x-10$, corrected upto four significant figures. **8**

OR

e) What is "Finite Differences"? Describe the different types of finite differences with suitable examples. **8**

f) From the following table of values find y(2.7) **8**

x	2.5	3.0	3.5	4.0	4.5
y(x)	9.75	12.45	15.70	19.52	23.75

Either:

4. a) Given the table of values as: **8**

x	2.5	3.0	4.5	4.75	6.0
y(x)	8.85	11.45	20.66	22.85	38.60

Find y(3.5) by using Divided Difference formula.

b) Approximate $\int_2^4 \dfrac{x}{x-1}dx$ using trapezoidal rule and Simpson's $\frac{1}{3}$rd rule for four subintervals. **8**

OR

e) Give the table of values as: **8**

x	0	1	2	3
y(x)	0	2	8	27

Find y(2.5)

f) Given $\frac{dy}{dx} = xy$ with $y(1) = 5$. Then find the solution correct to three decimal places in the interval [1, 1.5] for step size h = 0.1 using Range-Kutta second order method. **8**

5. Attempt all of the following. **4**

a) Perform the indicated operation and write the answer in standard form:
   i) (4 - 5i) (12 + 11i),
   ii) 8i(10 + 2i),
   iii) $\frac{7-i}{2+10i}$
   iv) (1 + 4i) – (-16 + 9i)

b) Explain the following terms with suitable examples; **4**
   i) Poles
   ii) Branch points

c) Find the root of the equation $x^3 + x^2 - 1 = 0$ on the interval (0, 1) corrected upto four significant figures using iteration method. **4**

d) Deduce the formula for linear least squares. **4**

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