M.Sc. - II (Mathematics) (New CBCS Pattern) Semester-IV

PSCMTH16 - Dynamical Systems

P. Pages : 2 **GUG/W/24/13767**

Time : Three Hours * 5 4 4 4 * Max. Marks : 100

Notes : 1. Solve **all five** questions.
2. All questions carry equal marks.

UNIT-I

1. a) Find a Lipschitz constant for $f(x) = x^{1/3}, -1 \le x \le 1$. **10**

b) If $f : W \to E$ is locally Lipschitz & $A \subset W$ is a compact (closed and bounded) set, then prove that f / A is Lipschitz. **10**

OR

c) Let $W \subset E$ be open, let $f : W \to E$ be a C^1 map. Let $y(t)$ be a solution on a maximal open interval $J = (\alpha, \beta) \subset R$ with $\beta < \infty$. Then prove that given any compact set $K \subset W$, there is some $t \in (\alpha, \beta)$ with $y(t) \notin K$. **10**

d) Prove that $\underset{t}{\oint}$ sends U on to an open set V and $\underset{-t}{\oint}$ is defined on V and sends V on to U. The composition $\underset{-t}{\oint}\underset{t}{\oint}$ is the identity map of U, the composition $\underset{t}{\oint}\underset{-t}{\oint}$ is the identity map of V. **10**

UNIT-II

2. a) Discuss the motion of pendulum moving in a vertical plane as an example of non-linear sink. **10**

b) Prove that E^* is isomorphic to E and thus has the same dimension. **10**

OR

c) Let E be a real vector space with an inner product and let T be a self adjoint operator on E. Then prove that the eigen values of T are real. **10**

d) There exists $\delta > 0$ such that if U is the closed ball $B\,\delta(0) \subset W$, then prove that for all $z = (x, y) \in C \bigcap U$. **10**

a) $\langle x, f_1(x, y)\rangle - \langle y, f_2(x, y)\rangle > 0$ if $x \ne 0$ and

b) There exists $\alpha > 0$ with $\langle f(z), z\rangle \ge \alpha |z|^2$

UNIT-III

3. a) Prove that a non-empty compact limit set of a C^1 planar dynamical system, which contains no equilibrium point, is a closed orbit. **10**

b) Let $y \in L_w(x) \cup L_\alpha(x)$. Then prove that the trajectory of y crosses any local section at not more than one point. **10**

OR

c) Let r be a closed orbit enclosing an open set U contained in the domain W of the dynamical system. Then prove that U contains an equilibrium. **10**

d) Prove that every trajectory of the Volterra-Lotka equations $x' = (A - By)x$, $y' = (Cx - D)y$, $A, B, C, D > 0$ is a closed orbit (except the equilibrium z and the co-ordinate axes). **10**

UNIT-IV

4. a) Let $g : S_0 \to S$ be a Poincare map for V. Let $x \in S_0$ be such that $\lim_{n\to\infty} g^n(x) = 0$. Then prove that $\lim_{t\to\infty} d(\phi_t(x), V) = 0$. **10**

b) Prove that let $\bar{x}$ be a fixed point of a discrete dynamical system $g : W \to E$. If the eigen values of $Dg(\bar{x})$ are less than 1 in absolute value, $\bar{x}$ is asymptotically stable. **10**

OR

c) Let $W \subset E$ be open and let $f : W \to E$ be C^r, $1 \le r \le \infty$. Then prove that the flow $\oint : \Omega \to E$ of the differential equation $x' = f(x)$ is also C^r. **10**

d) Let $A : J \to L(E)$ be a continuous map from an open interval J to the space of linear operators on E. Let $(t_0, u_0) \in J \times E$. Then prove that the initial value problem $x' = A(t)x$, $x(t_0) = u_0$, has a unique solution on all of J. **10**

5. a) Explain dynamical system with example. **5**

b) Define : (i) Stable equilibrium (ii) Asymptotically stable **5**

c) Define monotone sequences in planer dynamical systems. **5**

d) Explain asymptotic stability of closed orbits. **5**

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