B.Sc. (CBCS Pattern) Semester-III

USMT-05 - Mathematics-I : Real Analysis

P. Pages : 2 **GUG/W/24/11612**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT-I

1. a) Prove that, if limit of sequence $\langle s_n \rangle$ exist then it is unique. **6**

b) Find the limit of the sequence $\langle s_n \rangle$ where **6**

$$s_n = \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + ... + \frac{1}{(n+n)^2}$$

OR

c) Prove that, if $\lim_{n\to\infty} s_n = \ell$ then $\lim_{n\to\infty}\left[\frac{s_1 + s_2 + --- + s_n}{n}\right] = \ell$ **6**

d) Prove that, every convergent sequence of real number is a Cauchy sequence. **6**

UNIT - II

2. a) Prove that, if the series Σx_n and Σy_n converges then the series $\Sigma(x_n + y_n)$ converges and $\Sigma(x_n + y_n) = \Sigma x_n + \Sigma y_n$. **6**

b) Show that the series $1 + \frac{1}{2!} + \frac{1}{3!} + --- + \frac{1}{n!} + ---$ is convergent. **6**

OR

c) Let Σx_n be a positive term series such that $\lim_{h\to\infty} \frac{x_{n+1}}{x_n} \ell$ then prove that the series Σx_n. **6**

i) Converges if $\ell < 1$ ii) Diverges if $\ell > l$

iii) the test fails if $\ell = l$

d) Test the convergence of series. $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + ...$ **6**

UNIT - III

3. a) Show that the function $d : R \times R \to R$ defined by $d(x, y) = \left|x^2 - y^2\right|, \forall x, y \in R$ is a pseudo metric on R and is not a metric on R. **6**

b) Prove that a set A is open if and only if its complement is closed. **6**

OR

c) Let X be a metric space, if $\{x_n\}$ and $\{y_n\}$ are sequences in X such that $x_{n \to x}$ and $y_{n \to y}$ then show that $d(x_n, y_n) \to d(x, y)$. **6**

d) If A and B are subsets of a metric space X then prove that $\operatorname{int}(A \cap B) = (\operatorname{int} A) \cap (\operatorname{int} B)$ **6**

UNIT - IV

4. a) Show that any constant function defined on a bounded closed interval is integrable. **6**

b) Prove that, If ‘f’ is monotonic in [a, b] then it is integrable on [a, b]. **6**

OR

c) Let the function f and g be integrable on [a, b] then prove that **6**

$$(f+g) \in R[a,b] \text{ and } \int_a^b (f+g)(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

d) If f is continuous on [a, b] and $\left|f(x)\right| \le |<, \forall x \in [a,b]$ where k is a constant then prove that **6**

$$\left|\int_a^b f(x)\,dx\right| \le k(b-a)$$

5. Solve **any six.**

a) Evaluate $\lim s_n$ if $s_n = \dfrac{2+3.10^n}{3+4.10^n}$ **2**

b) Show that $\lim\limits_{n \to \infty} \dfrac{\sin n}{n} = 0$ **2**

c) Test the convergence of the series Σx_n where $x_n = \dfrac{n+1}{n+2}$ **2**

d) Test the convergence of the series. **2**

$\Sigma \dfrac{1}{\sqrt{n}} = 1 + \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{3}} + \ldots\ldots\ldots\ldots$ by P-series test

e) Prove that, a finite point set has no limit points. **2**

f) Write definition of- **2**

i) Open set ii) Closed set

g) For any partition P, prove that $L(p, f) \le U(P, F)$ **2**

h) Define Darboux’s upper sum and lower sum. **2**

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