B.Sc. (CBCS Pattern) Semester-V

USMT09 DSE-I - Mathematics Paper-I - Linear Algebra

P. Pages : 2 **GUG/W/24/13115**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT-I

1. a) Let R^+ be the set of all positive real numbers. Define the operations of addition $\oplus$ and scalar multiplication $\otimes$ as follows. **6**

$u \oplus v = uv,\ \forall u, v \in R^+$ and $\alpha \otimes u = u^\alpha,\ \forall u \in R^+$ and $\alpha \in R$ prove that R^+ is a real vector space.

b) Prove that $U = \{(x_1, x_2, x_3) \in V_3 \mid x_1 + x_2 = x_3\}$ is a subspace of V_3. **6**

OR

c) Let $v_1, v_2, \ldots\ldots, v_n$ be n vectors of a vector space V(F). **6**
Prove that

i) $[v_1, v_2, \ldots\ldots, v_n] = [\alpha_1 v_1, \alpha_2 v_2, \ldots\ldots, \alpha_n v_n],\ \alpha_i (70) \in F,\ \forall i$

& ii) $[v_1, v_2] = [v_1 - v_2, v_1 + v_2]$

d) Let $S = [(1,-1,0),(1,0,2)]$, $T = [(0,1,0),(0,1,2)]$. Determine the subspaces $S \cap T$ and $S + T$. **6**

UNIT-II

2. a) Let U and V be vector spaces over the same field F. Then prove that a function $T : U \to V$ is linear if and only if $T(\alpha u + \beta v) = \alpha T(u) + \beta T(v),\ \forall\ \alpha, \beta \in F$ and $u, v \in V$. **6**

b) Let a mapping $T : V_2 \to V_2$ be defined by $T(x, y) = (x', y')$, where $x' = x\cos\theta - y\sin\theta$, $y' = x\sin\theta + y\cos\theta$. Show that T is a linear map. **6**

OR

c) Let $T : U \to V$ be a linear map and U is finite dimensional vector space. **6**
Then prove that $\dim R(T) + \dim N(T) = \dim U$.

d) Prove that a linear map $T : V_3 \to V_3$ defined by $T(e_1) = e_1 - e_2$, $T(e_2) = 2e_2 + e_3$, $T(e_3) = e_1 + e_2 + e_3$ is neither 1-1 nor onto. **6**

UNIT-III

3. a) Let V be a finite dimensional vector space over F. Then prove that (i) $\dim V = \dim \hat{V}$ and (ii) for $v (\neq 0) \in V, \exists F \in \hat{V}$ such that $F(V) \neq 0$. **6**

b) Let V be the finite dimensional vector space over F. Then prove that $v \approx \hat{\hat{V}}$. **6**

OR

c) If f and g are in $\hat{V}$ the dual of a vector space V such that $F(V) = 0$ implies $g(v) = 0$, prove that $g = \lambda f$ for some $\lambda \in F$. **6**

d) For a finite dimensional vector space V, show that $\tilde{V}$ is a separating family. **6**

UNIT-IV

4. a) In F^n, for $u = (\alpha_1, ..., \alpha_n)$ and $v = (\beta_1, ..., \beta_n)$, define $(u, v) = \alpha_1 \bar{\beta}_1 + \alpha_2 \bar{\beta}_2 + ... + \alpha_n \bar{\beta}_n$, where $F = C$, show that this defines an inner product. **6**

b) Let $\{x_1, x_2, ..., x_n\}$ be an orthogonal set. Then prove that **6**

i) $\|x_1 + x_2 + ... + x_n\|^2 = \|x_1\|^2 + \|x_2\|^2 + ... + \|x_n\|^2$

& ii) Every orthonormal set is LT.

OR

c) If V is a finite dimensional inner product space and if w is a subspace of v. Then prove that $V = W + W^{\perp}$. More particularly, V is the direct sum of W and $W^{\perp}$. **6**

d) Using Gram-Schmidt process, orthonormalizes the set of vectors $\{(1, 0, 1, 0), (1, 1, 3, 0), (0, 2, 0, 1)\}$ of V_4. **6**

5. Solve **any six.**

a) Sets U and W are defined by $U = \{(x_1, x_2) \in V_2 | x_1 \geq 0\}$, $W = \{(x_1, x_2) \in V_2 | x_1 \leq 0\}$. Is $U \cap W$ a subspace of V_2? **2**

b) Define Basis of Vector Space. **2**

c) Define Linear Transformation. **2**

d) Show that $T : R^2 \to R^2$ defined by $T(x, y) = (x + 1, y + 2)$, is not a linear map. **2**

e) Let U and W be subspace of V over F. Then prove that $U \subseteq W \Rightarrow A(W) \subseteq A(U)$. **2**

f) Let V is finite dimensional and $v_1 \neq v_2$ are in V. Prove that there is an $f \in \hat{V}$ such that $f(v_1) \neq f(v_2)$. **2**

g) Define Inner product space. **2**

h) Define standard inner product. **2**

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