M.Sc. - I (Mathematics) (New CBCS Pattern) Semester-II

PSCMTHT10A : Differential Geometry

P. Pages : 2 **GUG/W/24/13750**

Time : Three Hours * 5 4 2 7 * Max. Marks : 100

Notes : 1. Solve all the questions.
2. Each questions carry equal marks.

UNIT- I

1. a) Show that the notion of r-equivalence of representations of a surface is an equivalence relation. **10**

b) Show that the equation of the normal N at a point P on the surface $r = r(u, v)$ is $R = r + a(r_1 \times r_2)$. **10**

OR

c) On the paraboloid $x^2 - y^2 = z$ find the orthogonal trajectories of the section by the planes z = const. **10**

d) Show that the two directions given by $Pdu^2 + 2Qdudv + Rdv^2 = 0$ are orthogonal on a surface iff $ER - 2QF + GP = 0$. **10**

UNIT – II

2. a) State & Prove that necessary & sufficient condition for a curve $u = u(t), v = v(t)$ on a surface $r = r(u, v)$ to be geodesic. **10**

b) Prove that the curves of the family $\frac{v^3}{u^2} = \text{constant}$ are geodesics on the surface with a metric $v^2du^2 - 2uvdudv + 2u^2du^2,\ u > 0,\ v > 0$. **10**

OR

c) If a geodesic on a surface of revolution cuts the meridian at a constant angle then show that the surface is a right cylinder. **10**

d) Show that any curve $u = u(t),\ v = v(t)$ on a surface $r = r(u, v)$ is a geodesic iff the principal normal at every point on the curve is normal to the surface. **10**

UNIT – III

3. a) Obtain the formula for normal curvature K_n of a curve at a point P of a surface. **10**

b) Find the second fundamental form for the general surface of revolution. **10**

OR

c) Show that the conditions for an elliptic, parabolic or hyperbolic point are independent of the particular parametric representation. **10**

d) Find the asymptotic lines on the catenoid of revolution $u = \cosh\left(\frac{z}{c}\right)$. **10**

UNIT – IV

4. a) State & prove that Weingarten equations. **10**

b) If N is the surface normal then show that $N_1 \times N_2 = \frac{LN - M^2}{H} N$. **10**

OR

c) Derive the second of Codazzi equations given by **10**

$$\frac{\partial M}{\partial v} - \frac{\partial N}{\partial u} = L \left\lceil \begin{matrix} 1 \\ 22 \end{matrix} + M \left(\left\lceil \begin{matrix} 2 \\ 22 \end{matrix} - \left\lceil \begin{matrix} 1 \\ 12 \end{matrix} \right) - N \right\lceil \begin{matrix} 2 \\ 12 \end{matrix} \right.$$

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d) Given the differential forms $I = du^2 + \cos^2 u\, dv^2$, $II = du^2 + \cos^2 u\, dv^2$. Find the surfaces for which I & II are the first & second fundamental forms. **10**

5. a) Define the orthogonal trajectories. **5**

b) Give the definitions of : **5**
i) geodesic disc ii) geodesic parallels.

c) Find L, M, N for a sphere : **5**
$r = (a\cos u\cos v,\ a\cos u\sin v, a\sin u)$.

d) Define the parallel surfaces & state one example. **5**

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