M.Sc. - II (Mathematics) (NEP Pattern) - Semester-III

# 03NEPMATH04.1 - Elective Paper-IV : Advanced Topics in Operations Research

P. Pages : 3 **GUG/W/24/16016**

Time : Three Hours * 7 3 2 0 * Max. Marks : 80

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Notes : 1. Solve the five questions.
2. Each question carries equal marks.

## UNIT – I

**1.** a) Solve the following integer programming problem using the cutting-plane algorithm: **8**

Maximize $z = 3x_1 + x_2 + 3x_3$ subject to the constraints

$-x_1 + 2x_2 + x_3 \le 4,\ \ 4x_2 - 3x_3 \le 2, x_1 - 3x_2 + 2x_3 \le 3,$

$x_1, x_2$ and $x_3$ all are non - negative integers.

b) Discuss the fractional cut method-mixed integer LPP. **8**

**OR**

c) Find the optimum integer solution to the following L.P.P.: **8**

Maximize $z = x_1 + 4x_2$ subject to the constraints,

$2x_1 + 4x_2 \le 7,\ 5x_1 + 3x_2 \le 15,\ x_1, x_2 \ge 0$ & are integers.

d) Use branch and bound method to solve the following L.P.P.: **8**

Minimize $z = 4x_1 + 3x_2$ subject to the constraints,

$5x_1 + 3x_2 \ge 30,\ \ x_1 \le 4,\ \ x_2 \le 6,\ \ x_1, x_2 \le 0$ and are integers.

## UNIT – II

**2.** a) Solve the following linear goal programming problem graphically: **8**

Find $x_1$ and $x_2$ so as to,

Minimize $z = G_1\left(d_3^+ + d_4^+\right) + G_2 d_1^+ + G_3 d_2^- + G_4\left(d_3^- + \frac{3}{2} d_4^-\right)$

and satisfy the goals:

$G_1 : x_1 + x_2 + d_1^- + d_1^+ = 40$

$G_2 : x_1 + x_2 + d_2^- - d_2^+ = 100$

$G_3 : x_1 + d_3^- - d_3^+ = 30$

$G_4 : x_2 + d_4^- - d_4^+ = 15$

$x_i, d_i^-, d_i^+ \ge 0,\ \ \text{for all}\ \ i = 1, 2, 3, 4$

The goals have been listed in order of priority.

b) Explain the simplex method for goal programming problem. **8**

**OR**

c) Use revised simplex method to solve the LPP: **8**
Minimize $z = x_1 + x_2$ subject to the constraints:
$x_1 + 2x_2 \geq 7,\ \ 4x_1 + x_2 \geq 6,\ \ x_1, x_2 \geq 0$

d) Solve the following L.P.P.: **8**
Maximize $z = \dfrac{5x_1 + 3x_2}{5x_1 + 2x_2 + 1}$ subject to the constraints,
$3x_1 + 5x_2 \leq 15,\ \ 5x_1 + 2x_2 \leq 10$ and $x_1, x_2 \geq 0$

**UNIT – III**

**3.** a) Explain the processing of 2 jobs through k machines. **8**

b) Use graphical method to find the minimum elapsed total time sequence of 2 jobs and 5 machines, when we are given the following information. **8**

| | | Machines | | | | |
|---|---|---|---|---|---|---|
| Job 1 | sequence : | A | B | C | D | E |
| | Time (in hours) : | 2 | 3 | 4 | 6 | 2 |
| Job 2 | sequence : | C | A | D | E | B |
| | Time (in hours) : | 4 | 5 | 3 | 2 | 6 |

**OR**

c) A supermarket has two girls serving at the counters. The customers arrive in a Poisson fashion at the rate of 12 per hour. The service time for each customer is exponential with mean 6 minutes. Find (i) the probability that an customer has to wait for service, (ii) the average number of customers in the system, and (iii) the average time spent by a customer in the super market. **8**

d) Explain the power supply model. **8**

**UNIT – IV**

**4.** a) Obtain the necessary & sufficient conditions for the optimum solutions of the **8**
NLPP: Minimize $z = f(x_1, x_2) = 3e^{2x_1+1} + 2e^{x_2+5}$
subject to the constraints: $x_1 + x_2 = 7$ & $x_1, x_2 \geq 0$.

b) Solve: Maximize $z = 3.6x_1 - 0.4x_1^2 + 1.6x_2 - 0.2x_2^2$ **8**
Subject to the constraints: $2x_1 + x_2 \leq 10$ and $x_1, x_2 \geq 0$.

**OR**

c) Solve graphically the following NLPP: **8**

Maximize $z = 2x_1 + 3x_2$ subject to the constraints,

$x_1.x_2 \leq 8$, ${x_1}^2 + {x_2}^2 \leq 20$ and $x_1, x_2 \geq 0$.

Verify that the Kuhn-Tucker conditions hold for the maxima you obtain.

d) Use Beale's method to solve the following NLPP: **8**

Minimize $z = 6 - 6x_1 + 2{x_1}^2 - 2x_1x_2 + 2{x_2}^2$ subject to the constraints,

$x_1 + x_2 \leq 2$ and $x_1, x_2 \geq 0$

**5.** a) State the 7 steps of fractional cut method for all integer LPP. **4**

b) Write the formulation of linear goal programming problem. **4**

c) State the basic terms used in sequencing. **4**

d) Explain non-linear programming problem. **4**

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