M. Tech. Mechanical Engineering Design (CBCS Pattern) Semester-I

MED11 - Advanced Engineering Mathematics

P. Pages : 2 **GUG/W/24/14186**

Time : Three Hours * 5 6 8 4 * Max. Marks : 70

Notes : 1. All questions carry equal marks.
2. Use of Non programmable calculator is permitted.

1. a) Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cdot \sin x$ **7**

b) Show that $\int_0^x x^{n+1} J_n(x)\,dx = x^{n+1} J_{n+1}(x)$, if $n > -1$. **7**

OR

2. a) Show that- **7**

i) $P_n(-x) = (-1)^n P_n(x)$, ii) $P_n(-1) = (-1)^n$

b) Express in terms of Legendre's polynomials. **7**

i) $1 + x - x^2$ ii) $x^3 + 1$

3. a) Solve $\frac{dy}{dx} = x^2 y - 1$, given $y(0) = 1$, find $y(0.1)$. **7**

b) Solve $\frac{dy}{dx} = \frac{y - x}{y + x}$, given y(0) = 1, by Picard's method and find y For $x = 0.1$. **7**

OR

4. a) Using Euler's modified method, solve the equation $\frac{dy}{dx} = x + \left|\sqrt{y}\right|$, given $y = 1$, when $x = 0$ for range $0 \le x \le 0.6$ Taking h = 0.2. **7**

b) Solve the following equation by Runge-Kutta method. **7**

$\frac{dy}{dx} = x - 2y$, given $y(0) = 1$, find $y(0.2)$.

5. a) Use modified Euler's method to solve the equation. **7**

$\frac{dy}{dx} = x + y$ for a given that $y(0) = 1, h = 0.05$.

b) Solve the following by Runge-Kutta method. **7**

$\frac{dy}{dx} = 3x + y^2, y(0) = 1$, find $y(0.2)$.

OR

6. a) If $\frac{dy}{dx} = \frac{1}{2}\left(y^2 + xy^2\right), y(0) = 1$, find the series solution upto Four terms by Taylor's series method and find y (0.1). **7**

b) Find the approximate value of y when $x = 0.1$, if $\frac{dy}{dx} = x - y^2$. **7**

7. a) Solve $\frac{dy}{dx} = x^2 + y$, given $y(0) = 1$, find $y(0.1)$ by using modified Euler's method. **7**

b) Solve $y' = x - y^2, y = 0$. When x = 0, find y (1) by using Milne's predicted method. **7**

OR

8. a) Solve $\frac{dy}{dx} = y - \frac{2x}{y}, y(0) = 1, x = 0$ find y (1) by using Runge-Kutta method. **7**

b) Using Picard's Method, find approximate value of y and z corresponding to x = 0.1, x=0.2, given that $\frac{dy}{dx} = 2x + 2, \frac{dz}{dx} = 3xy + x^2z, y(0) = 2, z(0) = 0$. **7**

9. a) Fit a straight line to the following set of data points: **7**

x	1	2	3	4	5
y	3	4	5	6	8

b) Find the best-fit values of a and b so that y = a + bx fits the data given in the table. **7**

x	0	1	2	3	4
y	1	1.4	3.3	4.5	6.3

OR

10. a) Find least squares polynomials approximation of degree two to the data. **7**

x	0	1	2	3	x
y	-4	-1	4	11	20

b) Fit a second-degree parabola to the following data. **7**

x	1	2	3	4	5	6	7	8	9	10
y	124	129	140	159	228	289	315	302	263	210

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