

M.Sc.(Physics) (NEP Pattern) Semester-I  
**NEP-235 / 01MSCPH3 - DSC Paper-III - Mathematical Physics**

P. Pages : 2

Time : Three Hours



**GUG/W/24/15136**

Max. Marks : 80

**Either:**

1. a) Find the eigen values and eigen vectors of matrix. 8

$$A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$$

- b) Find the  $A^{-1}$  of the matrix by using Cayley-Hamilton theorem. 8

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

**OR**

- e) State and prove Cayley-Hamilton theorem. 8

- f) Explain: 8

i) Vector space

ii) Inner product space

**Either:**

2. a) Find the Fourier series of the function 8

$$F(x) = \begin{cases} -1 & \text{for } -\pi < x < 0 \\ 1 & \text{for } 0 < x < \pi \end{cases}$$

- b) Find the inverse Laplace transform of 8

$$\frac{s^2 - 1}{(s^2 + 1)^2}$$

**OR**

- e) Write properties of Laplace transform. 8

- f) State and prove Fourier integral theorem. 8

**Either:**

3. a) Express,  $F(x) = 4x^3 - 2x^2 - 3x + 8$  in terms of Legendre polynomials. 8

- b) Solve the differential equations. 8

i)  $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x} - \log 2$

ii)  $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = e^x \cdot \cosh 2x$

**OR**

- e) Solve the differential equation by power series solution. 8

$$2x \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - (x+1)y = 0$$

- f) Prove that

i)  $xJ'_n = n \cdot J_n - xJ_{n+1}$  4

ii)  $x \cdot J'_n = -nJ_n + xJ_{n-1}$  4

**Either:**

4. a) Prove that  $(y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$  is both solenoidal and irrotational. 8

- b) What do you mean by Symmetric and antisymmetric tensor? Show that any second order tensor can be expressed as the sum of symmetric and skew symmetric tensors. 8

**OR**

- e) Find curl and divergence of  $\vec{V}$  8

$$\vec{V} = \frac{x \vec{i} + y \vec{j} + z \vec{k}}{\sqrt{x^2 + y^2 + z^2}}$$

- f) i)  $\text{grad} \left( \vec{f} \cdot \vec{g} \right) = \vec{f} \times \text{curl} \vec{g} + \vec{g} \times \text{curl} \vec{f} + \vec{f} \Delta \vec{g} + \vec{g} \Delta \vec{f}$  4

ii)  $\text{curl} \left( \vec{f} \times \vec{g} \right) = \vec{f} \text{div} \vec{g} - \vec{g} \text{div} \vec{f} + \vec{g} \Delta \vec{f} - \vec{f} \Delta \vec{g}$  4

5. a) Answer all of the followings. 4

a) Find  $A^{-1}$  of matrix,  $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 1 & 2 \end{bmatrix}$

- b) Find the Fourier sine transform of  $F(x) = e^{-ax}$ , for  $0 < x < \infty$  4

- c) What is generating of Bessel function? Show that  $2J'_n = J_{n-1} - J_{n+1}$  4

- d) Define divergence of a vector and give its physical meaning. 4

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