M.Tech. Electronics & Communication Engineering (CBCS Pattern) Semester-I

PECS11 - Probability Theory and Stochastic Processes

P. Pages : 2 **GUG/W/24/10978**

Time : Three Hours Max. Marks : 70

Notes :
1. All questions carry marks as indicated.
2. Due credit will be given to neatness and adequate dimensions.
3. Assume suitable data wherever necessary.
4. Illustrate your answers wherever necessary with the help of neat sketches.

1. a) A fair coin is tossed twice and let the random variable X represent number of heads. Find $F_X(X)$ **7**

b) State and explain Baye's theorem with suitable example. **7**

2. a) Determine Binomial distribution for which mean is 4 and variance is 3. **7**

b) A random variable X has the density function. **7**

$$F(X) = k \cdot e^{-3x}, \quad x > 0$$
$$= 0 \qquad , x \le 0$$

Find constant K, mean and variance of X.

3. a) If 3% of electric bulbs manufactured by a company are defective find the probability that in a sample of 100 bulbs: **7**

i) exactly 2

ii) at most 2

iii) at least 2 bulbs are defective

b) Explain hypergeometric distribution. Evaluate mean and variance for it. **7**

4. A person writes 'n' letters and addresses 'n' envelopes, then one letter is placed into each envelope. What is the probability that at least one letter will reach its correct destination? What if $n \to \infty$? **14**

5. a) Explain Log-Normal distribution. The current gain of certain transistor is measured in units which make it equal to the logarithm of Io/Ii; the ratio of output to the input current of this logarithm is normally distributed with $\mu = 2$ and $\sigma^2 = 0.1$. Find the probability that Io/Ii will take on a value between 6.1 and 8.2. **8**

b) Explain mean and variance of Rayleigh distribution. **6**

6. a) The joint probability distribution of (x, y) is **8**

y \ x	-1	0	1
-1	0.0	0.1	0.1
0	0.2	0.2	0.2
1	0.0	0.1	0.1

Find $E\left[\left(\frac{y}{x}\right)=-1\right]$ and $\gamma\left[\left(\frac{y}{x}\right)=-1\right]$.

b) Prove that $\sigma_{XY}=E(XY)-\mu_X\mu_Y$. **6**

7. a) A random process defined by $X(t)=A\cos(w_o t+\theta)$ where A and w_o are constants, θ is uniformly distributed random variable in the interval $(0,2\pi)$. Show that X(t) is a wide sense stationary process? **7**

b) Prove that the process X(t) is strict sense stationary iff the joint density f(a, b) of random variables a and b has circular symmetry, i.e. if $f(a,b)=f\sqrt{a^2+b^2}$. **7**

8. a) A stationary process V(t) with auto correlation $R_{VV}(t)=q\cdot\delta(t)$ (white noise) is applied at t = 0 to a linear system with $h(t)e^{-ct}.U(t)$. Prove that the auto correlation of the resulting output $y(t)$ is $R_{YY}(t_1,t_2)=\frac{q}{2c}\left(1-e^{-2ct_1}\right).e^{-c|t_1-t_2|}$ **8**

b) Explain Markov process in detail. **6**

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