B.Sc.- II (CBCS Pattern) Semester-IV

USMT-07 - Mathematics Paper-I - Algebra

P. Pages : 2 **GUG/W/24/12014(S)**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all **five** questions
2. All questions carry equal marks.

UNIT – I

1. a) Show that the set of cube roots of unity forms an abelian group with respect to the usual multiplication of numbers. **6**

b) Prove that in a group G. The linear equations ax = b and ya = b have unique solutions for x and y in $G, \forall a, b \in G$. **6**

OR

c) Prove that a non-empty subset H of the group G is a subgroup of G if and only if- **6**
(i) $\forall a, b \in H \Rightarrow ab \in H$ and (ii) $\forall a \in H \Rightarrow a^{-1} \in H$.

d) For $S = \{1, 2, 3, \ldots\ldots, 9\}$ and $a, b \in A(s)$ then compute $a^{-1}ba$, where **6**
$a = (1, 3 5)(1, 2)$, $b = (1, 5\ 7\ 9)$.

UNIT – II

2. a) Prove that a subgroup N of G is a normal subgroup of G if and only if the product of two right cosets of N in G is again a right coset of N in G. **6**

b) Suppose that N and M are two normal subgroups of G and that $N \cap M = \{e\}$. **6**
Show that for any $n \in N, m \in M$, $nm = mn$.

OR

c) If H is a subgroup of G and N is a normal subgroup of G, show that $H \cap N$ is a normal subgroup of H. **6**

d) If G is a finite group and H is a subgroup of G, then prove that O(H) is a divisor of O(G). **6**

UNIT – III

3. a) Let G be any group, g a fixed element in G. Define $\phi : G \to G$ by $\phi(x) = gxg^{-1}$. **6**
Prove that ϕ is an isomorphism of G onto G.

b) Let N be a normal subgroup of G. Define mapping $\phi : G \to G / N$ such that **6**
$\phi(x) = Nx, \forall x \in G$ then prove that ϕ is a homomorphism of G onto G/N.

OR

c) If ϕ is a homomorphism of G into G' with kernel K, then prove that K is a normal subgroup of G. **6**

d) Let ϕ be a homomorphism of G onto G' with kernel K, then prove that $G/K \approx G'$. **6**

UNIT – IV

4. a) If in a ring R, $x^3 = x, \forall x \in R$, then show that R is commutative. **6**

b) If R is a ring with zero element O, then prove that $\forall a, b \in R$ **6**

i) $a0 = 0a = 0$ ii) $a(-b) = (-a)b = -(ab)$

OR

c) Prove that intersection of two subrings is a subring. **6**

d) Show that the ring $R = \left\{a + b\sqrt{2} / a, b \in Z\right\}$ is an integral domain under addition and multiplication. **6**

5. Attempt **any six.**

a) Prove that every element of a group G has a unique inverse in G. **2**

b) If G is a group such that $(ab)^2 = a^2b^2, \forall a, b \in G$, show that G must be abelian. **2**

c) Define left coset of H in G. **2**

d) Define normal subgroup of G. **2**

e) Define kernel of Homomorphism. **2**

f) If ϕ is an homomorphism of a group G into a group G', then prove that $\phi\left(x^{-1}\right) = \left(\phi(x)\right)^{-1}, \forall x \in G$. **2**

g) Let R be a ring. Prove that if $a, b \in R$ then $(a+b)^2 = a^2 + ab + ba + b^2$. **2**

h) Define Integral domain. **2**

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