M.Sc.(Mathematics) (NEP Pattern) - Semester-III

03NEPMATH02 - Partial Differential Equations

P. Pages : 2 **GUG/W/24/16014**

Time : Three Hours *7318* Max. Marks : 80

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT-I

1. a) Prove that singular integral of $f(x,y,z,p,q)=0$ satisfies the equations $f(x,y,z,p,q)=0$, $f_p(x,y,z,p,q)=0$ and $f_q(x,y,z,p,q)=0$. **8**

b) Find the general integral of $(x^2+y^2)p+2xyq=(x+y)z$. **8**

OR

c) Show that the equations $f=p^2+q^2-1=0$ and $g=(p^2+q^2)x-pz=0$ are compatible and find the one parameter family of common solutions. **8**

d) Find the complete integral of $(p^2+q^2)y-qz=0$ by Jacobs method. **8**

UNIT-II

2. a) Find the integral surface of the p.d.e. $(x-y)y^2p+(y-x)x^2q=(x^2+y^2)z$ through the curve $xz=a^2, y=0$. **8**

b) Solve the equation $xz_y-yz_x=z$ with the initial condition $z(x,0)=f(x), x\geq 0$. **8**

OR

c) Find the solution of the equation $z=\frac{1}{2}(p^2+q^2)+(p-x)(q-y)$ which passes through the x-axis. **8**

d) Find the integral surface of the equation $(2xy-1)p+(z-2x^2)q=2(x-yz)$ which passes through the line $x_0(s)=1,\ y_0(s)=0$ and $z_0(s)=s$. **8**

UNIT-III

3. a) Derive an equation governing small transverse vibrations of a string. **8**

b) Reduce the equation $u_{xx}-x^2u_{yy}=0$ to a canonical form. **8**

OR

c) Obtain D'Alembert's solution of the one dimensional wave equation which describes the vibrations of an infinite string. **8**

d) Prove that for the equation $Lu = u_{xy} + \frac{1}{4}u = 0$, the Riemann function is $v(x, y : \alpha, \beta) = J_0\left(\sqrt{(x-\alpha)(y-\beta)}\right)$, where J_0 denotes the Bessel's function of the first kind of order zero. **8**

UNIT-IV

4. a) Prove that: If $u(x, y)$ is harmonic in a bounded domain D and continuous in $\overline{D} = D \bigcup B$. Then u attains its maximum on the boundary B and D. **8**

b) Show that the solution of the Dirichlet problem is stable. **8**

OR

c) Show that the solution for the Dirichlet problem for a circle of radius a is given by Poisson integral formula. **8**

d) Solve $\nabla^2 u = 0, r < a$ subject to the boundary condition $\frac{\partial u}{\partial n} = \frac{\partial u}{\partial r} = f(\theta)$ on $r = a$, where $\int_0^{2\pi} f(\theta)\, d\theta = 0$. **8**

5, a) Eliminate the arbitrary function F from the equation $F(z - xy, x^2 + y^2) = 0$ and find the corresponding partial differential equation. **4**

b) Define: **4**

i) Semi-linear partial differential equation

ii) Quasi-linear partial differential equation

c) State Green's theorem. **4**

d) Prove that the solution of the Dirichlet problem, if it exists, is unique. **4**

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