

M.C.A. - I (2 Years) New CBCS Pattern Semester-I  
**PSMCAT104.2 - Elective-II Paper-IV : Discrete Mathematics**

P. Pages : 3

Time : Three Hours



**GUG/W/24/13639**

Max. Marks : 80

- Notes :
1. All questions are compulsory and carry equal marks.
  2. Draw neat and labelled diagram and use supporting data whenever necessary.
  3. Avoid vague answer and write specific answer related to questions.

**Either:**

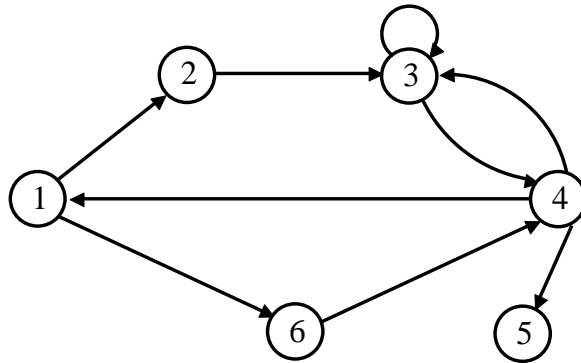
1. a) Construct the truth table for. 8
- i)  $(P \wedge Q) \vee (\neg P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$
  - ii)  $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R)$
- b) Show that the following are equivalent formula. 8
- i)  $P \vee (P \wedge Q) \Leftrightarrow P$
  - ii)  $P \vee (\neg P \wedge Q) \Leftrightarrow P \vee Q$

**OR**

- c) Define following terms with truth table. 8
- |                  |                           |
|------------------|---------------------------|
| i) Negation      | ii) Conjunction           |
| iii) Disjunction | iv) Conditional statement |
- d) Obtain conjunctive Normal form of formula s given by  $(\neg P \rightarrow R) \wedge (Q \leftrightarrow R)$ . 8

**Either:**

2. a) Let  $A = \{a, b\}$  &  $B = \{4, 5, 6\}$  Determine 8
- |                   |                  |
|-------------------|------------------|
| i) $A \times B$   | ii) $B \times A$ |
| iii) $A \times A$ | iv) $B \times B$ |
- b) Let R be the relation whose diagram is given below. 8



- i) Find a cycle starting at vertex 6.
- ii) Find  $M_R^2$ .

**OR**

d) Let  $A = \{1, 2, 3, 4\}$  and  $R = \{(2, 1), (2, 3), (3, 2), (4, 3)\}$ . Find the transitive closure of  $R$  using Warshall's Algorithm. 8

d) Let  $A = \mathbb{Z}^+$ , the set of positive integers and Let  $R = \{(a, b) \in A \times A \mid a \text{ divides } b\}$ . Is  $R$  Symmetric a asymmetric or antisymmetric? 8

**Either:**

3. a) Define the following 8  
 i) Partially Ordered set                      ii) House diagram with example  
 iii) Lattice                                      iv) Sub lattice

b) In a lattice prove that 8  
 i)  $(a * b) \oplus (a * c) \leq a * [b \oplus (a * c)]$   
 ii)  $(a \oplus b) * (a \oplus c) \geq a \oplus [b * (a \oplus c)]$

**OR**

c) Prove the following Boolean identities. 8  
 i)  $a \oplus (a' * b) = a \oplus b$   
 ii)  $a * (a' \oplus b) = a + b$   
 iii)  $(a * b) \oplus (a * b') = a$   
 iv)  $(a * b * c) \oplus (a * b) = (a * c)$

d) Prove that :  $a \cdot (b + c) = a \cdot b + a \cdot c$  . 8

**Either:**

4. a) Consider machine  $M$  where table is shown below. 8

	a	b	c
$S_0$	$S_0$	$S_0$	$S_0$
$S_1$	$S_2$	$S_3$	$S_2$
$S_2$	$S_1$	$S_0$	$S_3$
$S_3$	$S_3$	$S_2$	$S_3$

Draw the diagram and state transition function.

b) Let  $G$  be a group. Prove that each element  $a$  in  $G$  has only one inverse in  $G$ . 8

**OR**

c) Fill in the following table so that the binary operation  $*$ , is commutative. 8

$*$	a	b	c
a	b		
b	c	b	a
c	a		c

- d) Let  $G$  be the grammar

$$S \rightarrow aB \mid bA$$
$$A \rightarrow a \mid as \mid bAA$$
$$B \rightarrow b \mid bs \mid aBB$$

For the string 'aaabbabbba'.

Find :

- i) Leftmost derivation
- ii) Rightmost derivation
- iii) Parse tree

**5.** Solve all the questions.

- a) Prove that  $(P \rightarrow Q) \leftrightarrow (\neg P \rightarrow \neg Q)$  is a tautology.

- b) Let  $A = \{a, b\}$  and  $B = \{4, 5, 6\}$  Determine

- i)     $A \times B$   
ii)     $B \times A$   
iii)    $A \times A$   
iv)     $B \times B$

- c) Let  $s = \{a, b, c\}$ . Draw the Hasse diagram of  $(P(s), \subseteq)$

- d) Write a short note on Moore Machine.

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