B.Sc. (CBCS Pattern) Semester-V

USMT10 DSE-II - Mathematics-II - Mechanics

P. Pages : 3

GUG/W/24/13116

Time : Three Hours

* 4 9 5 3 *

Max. Marks : 60

Notes : 1. Solve all the **five** questions.
2. All questions carry equal marks.

UNIT - I

1. a) The velocities of a particle along and perpendicular to the radius vector from a fixed origin are λr^2 and $\mu\theta^2$. Show that the equation to the path is $\frac{\lambda}{\theta} = \frac{\mu}{2r^2} + c$ and component accelerations are $2\lambda^2 r^3 - \mu^2 \frac{\theta^4}{r}$ and $\lambda u r \theta^2 + 2\mu^2 \frac{\theta^3}{r}$. **6**

b) A particle moves along a circle $r = 2a\cos\theta$ in such a way that its acceleration towards the origin is always zero prove that-
$\frac{dw}{dt} = -2w^2 \cot\theta$, where $w = \frac{d\theta}{dt}$. **6**

OR

c) Prove that if the tangential and normal acceleration of a particle describing a plane curve be constant throughout the motion, the angle ψ which the direction of motion turns in time t is given by $\psi = a\log(1+Bt)$. **6**

d) V_1 and V_2 are the velocities of a particle moving in SHM at distance X_1 and X_2 from the centric. Show that the time of complete oscillation is $2\pi\sqrt{\frac{x_1^2 - x_2^2}{V_2^2 - V_1^2}}$ **6**

UNIT - II

2. a) Prove that the centre of mass of the system of particles moves as if the external forces were acting on the mass of the system concentrated at the centre of mass. **6**

b) Prove that the system of particles $N^{-(e)} = \dot{\bar{L}}$, where $\bar{N}^{(e)}$ is the total external torque and $\bar{L}$ the total angular momentum. **6**

OR

c) Show that the total kinetic energy of the system is the sum of the kinetic energy of the total mass concentrated at the cente of mass G and the kinetic energy of the system about the centre of mass G. **6**

d) Prove that the total angular momentum of the system of particles about any point 0 is the sum of the angular momentum of the system concentrated at the centre of mass and the angular momentum of motion about the centre of mass. **6**

UNIT - III

3. a) Obtain Lagrange's equations of motion for the system of particles in the form. **6**

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{a}_j}\right) - \frac{\partial T}{\partial \dot{a}_j} = Q_j, \ j = 1, 2, ..., n \ .$$

b) Discuss the motion of a particle in a plane by using polar co-ordinate. **6**

OR

c) Show that Lagrange's equation of motion take the form $\left(\frac{\partial L}{\partial \dot{a}_j}\right)^{\bullet} - \frac{\partial L}{\partial a_j} + \frac{\partial R}{\partial \dot{a}_j} = 0$. when the frictional forces, acting on the system are derivable in terms of Rayleigh's dissipation function R. **6**

d) A bead is sliding on a uniformly rotating wire in a force free space. Show that the acceleration of the bead is $\ddot{r} = r\,w^2$, where w is the angular velocity of rotation. **6**

UNIT - IV

4. a) Prove that, for a central force field F, the path of a particle of mass m is given by **6**

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{h^2u^2}F\left(\frac{1}{u}\right), \ u = \frac{1}{r}.$$

b) Show that if a particle describes a circular orbit under the influence of an attractive central force directed towards point on circle, then the force varies as inverse fifth power of the distance. **6**

OR

c) Prove that the orbit of each plant is an ellipse with the sun at one focus. **6**

d) State and prove virial theorem. **6**

5. Solve **any six.**

a) Find the radial and transverse components of acceleration of a moving particle in a circular path of radius a. **2**

b) Of the radial and transverse velocities of a particle are always proportions to each other, show that the path is an equiangular spiral. **2**

c) If the total torque on the system is zero, then prove that the total angular momentum is conserved. **2**

d) Define centre of mass. **2**

e) Define generalized co-ordinates and velocities. **2**

f) State D'Alembert's Principle. **2**

g) Prove that the path of a particle in a central force field lies in one plane. **2**

h) State Kepler's third law of planetary motion. **2**

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