B.Sc. (CBCS Pattern) Semester-III

USMT-06 - Paper-VI - Mathematics-II : Set Theory and Laplace Transform

P. Pages : 3 **GUG/W/24/11613**

Time : Three Hours * 4 5 6 5 * Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT - I

1. a) Prove that $A\times(B\cup C)=[A\times B]\cup(A\times C)$ **6**

b) With the help of the Venn diagram, prove that- **6**

i) $(A\cup B)'=A'\cap B'$ and

ii) $(A\cap B)'=A'\cup B'$

OR

c) Show that on a family of sets the relation 'equality of sets' is an equivalence relation but 'a subset of' is not an equivalence relation. **6**

d) Prove that every infinite subset of a countable set is countable. **6**

UNIT - II

2. a) let $A,B\in\tilde{P}(U)$. Then prove that $\alpha\le\beta\Rightarrow {}^{\beta}A\subseteq {}^{\alpha}A$ and ${}^{\beta+}A\subseteq {}^{\alpha+}A,\ \forall\alpha,\beta\in[0,1]$. **6**

b) Prove that A fuzzy set F on the universal set R is converse if and only if- **6**
$F[\lambda x_1+(1-\lambda)x_2]\ge\min[F(x_1),F(x_2)]\forall x_1,x_2\in R$ and all $\lambda\in[0,1]$.

OR

c) For the fuzzy sets $\tilde{A}$ and $\tilde{B}$ in the universal set $U=\{2,3,4\}$ defined by **6**
$\tilde{A}=\frac{0.3}{2}+\frac{0.4}{3}+\frac{0.6}{4}$ and $\tilde{B}=\frac{0.2}{2}+\frac{0.7}{3}+\frac{0.1}{4}$ find the product $\tilde{A}\tilde{B}$.

d) Let $\tilde{A},\tilde{B}\in p(U)$. Then for all $a\in[0,1]$, show that **6**

i) $\tilde{A}=\tilde{B}$ if and only if ${}^{\alpha}\tilde{A}={}^{\alpha}\tilde{B}$

ii) $\tilde{A}=\tilde{B}$ if and only if ${}^{\alpha+}\tilde{A}={}^{\alpha+}\tilde{B}$.

UNIT - III

3. a) Find the LT of **6**

i) $(\sin t - \cos t)^2$ and

ii) $\sin t \cdot \cos 2t \cdot \cos 3t$

b) If $c_1, ...c_n$ are any constant and $F_1(t), ..., f_n(t)$ are functions whose Laplace transforms exist, then prove that- **6**

$$L[c_1 f_1(t) + ... + c_n f_n(t)] = c_1 L[f_1(t)] + ... + c_n L[f_n(t)]$$

OR

c) Find the Laplace transforms of the following functions- **6**

i) $e^{-3t}(2\cos 5t - 3\sin 5t)$

ii) $2e^t \sin 4t \cdot \cos 2t$.

d) If $L[f(t)] = F(s)$, then prove that $L[e^{at} f(t)] = f(s-a)$. **6**

UNIT - IV

4. a) If $L^{-1}[F(s)] = f(t)$ and $L^{-1}[G(s)] = g(f)$. Then prove that **6**

$$L^{-1}[F(s) \cdot G(s)] = \int_0^t f(u) g(t-u) du$$

b) Obtain the inverse Laplace transform of- **6**

$$\frac{s^2 + 2a}{(s^2 + 4)(s^2 + a)}$$

OR

c) Use Laplace transform to find solution of the following equations. **6**

$(D^2 + n^2)x = a\sin(nt + \alpha), x = Dx = 0$ at $t = 0$

d) Solve the equation $ty'' + (1-2t)y' - 2y = 0, y(0) = 1, y'(0) = 2$ **6**

5. Solve **any six.**

a) Define Indexed sets and power set of A. **2**

b) Let R be a relation from A={2, 4, 9} to B = {2, 3} defined by $R = \{(a,b)/a \in A, b \in B, \text{ a is divisible by b}\}$. Find the relation R. **2**

c) Define α-cut. **2**

d) Let $\tilde{A} = \left\{\frac{0}{a}, \frac{0}{b}, \frac{0.4}{c}, \frac{0.5}{d}, \frac{0.6}{e}, \frac{0.9}{f}\right\}$ in $U = \{a,b,c,d,e,f\}$. Find $\rho(\tilde{A})$. **2**

e) Find LT of $\sin^2 2t$. **2**

f) Find $L[\cos t \cdot \cos 2t]$. **2**

g) Find $L^{-1}\left[\frac{s+3}{s^2+1}\right]$ **2**

h) Define a convolution integral. **2**

****