

SE201 - Engineering Mathematics-III - Probability and Statistics

P. Pages : 3

Time : Three Hours

**GUG/W/24/13856**

Max. Marks : 80

- Notes : 1. All questions carry equal marks.
2. Use of Non-Programmable calculator is permitted.

1. a) Let X be a random variable giving the number of aces in a random draw of four cards from a pack of 52 cards. Find the probability function and the distribution function for X. **8**

- b) Find the distribution function for a r.v. X whose density function is **8**
- $$f(x) = \begin{cases} \frac{x}{2}, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$
- hence or otherwise find $P(1/2 < X < 3/2)$ and $P(X > 1)$.

OR

2. a) The joint probability function of two discrete random variable X and Y is given by- **8**

$$f(x, y) = \begin{cases} c(2x + y), & 0 \leq x \leq 2, 0 \leq y \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

Find:

- | | |
|--|-------------------------------|
| i) the constant c | ii) $E(x) E(y)$ |
| iii) $\text{Var}(x)$ and $\text{Var}(y)$ | iv) $\text{Cov}(X, Y)$ and P. |

- b) A random variable X has density function **8**

$$f(x) = \begin{cases} kx^2, & 1 \leq x \leq 2 \\ kx, & 2 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases} \text{ find}$$

- | | |
|---------------------|--------------------------------|
| i) Constant K | ii) $P(1 < x < 3)$ |
| iii) $P(2 < x < 3)$ | iv) The distribution functions |

3. a) The chance of doctor A will diagnose a disease X correctly is 60%. The chance that a patient will die by his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of doctor A who had diseases X died. What is the chance that his disease was diagnosed correctly. **8**

- b) Find first four moments about (i) origin and (ii) mean for the probability distribution **8**

$$f(x) = \begin{cases} \frac{4x(9-x^2)}{81}, & 0 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

OR

4. a) Let X and Y be random variables having joint density function 8

$$f(x, y) = \begin{cases} \frac{3x(x+y)}{5}, & 0 \leq x \leq 1, 0 \leq y \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

Find:

- i) $E(X)$ ii) $E(x^2)$
 iii) $E(x^2 + y^2)$ iv) $\text{Var}(X)$

- b) In a certain city, the daily consumption of water (in millions of liters) follows approximately a gamma. Distribution with $\alpha = 2$ and $\beta = 3$. If the daily capacity of the city is 9 million liters of water. What is the probability that on any given day the water supply is inadequate. 8

5. a) A random variable X has the density function given by $f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$ 8

find the coefficient of skewness and kurtosis.

- b) Find the coefficients of 8

- i) Skewness
 ii) Kurtosis for the distribution with density function

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

OR

6. a) Find the coefficient of correlation between the variables x and y and hence find the regression lines. 8

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

- b) Find the rank correlation coefficient to the following data. 8

x	65	63	67	64	68	62	70	66	68	67	69	71
y	68	66	68	65	69	66	68	65	71	67	68	70

7. a) By the method of least squares, find the straight line that best fits the following data 8

x	1	2	3	4	5
y	14	27	40	55	68

- b) Fit the curve $y = ae^{bx}$ to the following data 8

x	1	2	3	4	5	6
y	7.209	5.265	2.846	2.809	2.052	1.499

OR

8. a) Show that the lines of fit to the following data is given by $y = 0.7x + 11.285$ 8

x	0	5	10	15	20	25
y	12	15	17	22	24	30

- b) Employ the method of least squares to fit a parabola 8
 $y = a + bx + cx^2$ in the following data $(x, y) : (-1, 2), (0, 0), (0, 1), (1, 2)$

9. a) In 200 tosses of coin 115 heads and 85 tails were observed. Test the hypothesis that the coin is fair using a level of significance of (a) 0.05 (b) 0.01 (c) find the P value of the test. 8

- b) In a sample of 600 men from a certain city 450 are found smokers in another sample of 900 men from another city 450 are smokers do the data indicate that the cities are significantly different with respect to the habit of smoking among men. 8

OR

10. a) Out of 400 children, 150 are found to be under weight. Assuming the condition of simple sampling estimates the percentage of children who are underweight in and assign limits within which the percentage probably lies. 8

- b) One type of aircraft is found to developed engine troubles in 5 flights out of total of hundred and another type in 7 flights out of 200 total flights is there a significant difference in the two types of aircrafts so far as engine defects are concerned. 8
