M.Sc. (Part-II) (Mathematics) (New CBCS Pattern) Semester-III

PSCMTH13 - Core Course : Mathematical Methods

P. Pages : 3 **GUG/W/24/13757**

Time : Three Hours * 5 4 3 4 * Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) Define the Fourier transforms. If C_1 and C_2 are constants, then show that $F[c_1 f_1(x)+c_2 f_2(x)] = c_1 f[f_1(x)] + c_2 f[f_2(x)]$ **10**

b) i) If $F[f(x): x \to \xi] = F(\xi)$, then show that $F[f(ax): x \to \xi] = \frac{1}{a} F(\frac{\xi}{a})$ **10**

ii) If $F_s[f(x)] = F_s(\xi)$, then prove that $F_s[f(ax)] = \frac{1}{a} F_s\left(\frac{\xi}{a}\right)$

OR

c) If $F(\xi)$ is Fourier transform of f(x) in R, then show that Fourier transform of $f(x-a)$ is $e^{i\xi a} \cdot F(\xi)$. **10**

d) Show that $\int_0^\infty \frac{\cos px}{1+p^2} dp = \frac{\pi}{2} e^{-x}, \ x \ge 0$ **10**

UNIT – II

2. a) Let f(x) be continuous and $f'(x)$ be sectionally continuous on the interval $0 \le x \le a$, then prove that **10**

i) $\bar{f}_c\left[f'(x);\ x \to n\right] = (-1)^n f(a) - f(0) + \frac{n\pi}{a} \bar{f}_s(n), n \in z^*$

ii) $\bar{f}_s\left[f'(x);\ x \to n\right] = \frac{-n\pi}{a} \bar{f}_c(n), n \in N$

b) Let f(x) and $f'(x)$ be continuous and $f''(x)$ be sectionally continuous on $0 \le x \le a$. Then prove that **10**

i) $\bar{f}_c\left[f''(x); n\right] = -f'(0) + (-1)^n f'(a) - \frac{n^2\pi^2}{a^2} \bar{f}_c(n)$

ii) $\bar{f}_s\left[f''(x); n\right] = \frac{n\pi}{a} f(0) - (-1)^n \frac{n\pi}{a} f(a) - \frac{n^2\pi^2}{a^2} \bar{f}_s(n)$

OR

c) Solve the wave equation **10**

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2},\ 0 \le x \le a,\ t > 0$$

Satisfying the boundary condition $u(0,t) = u(a,t) = 0, t > 0$ and the initial condition $u(x,0) = \frac{4b}{a^2}x(a-x),\ \frac{\partial u(x,0)}{\partial t} = 0,\ 0 \le x \le a$ to determine the displacement u(x, t)

d) Find finite sine and cosine transforms of $f(x) = x^2,\ 0 < x < \pi$. **10**

UNIT – III

3. a) If $L[f_1(t);\ t \to p] = \overline{f}_1(p)$ and $L[f_2(t);\ t \to p] = \overline{f}_2(p)$ both exist and c_1, c_2 are constants, then slow that $L[c_1 f_1(t) + c_2 f_2(t); t \to p] = c_1 L[f_1(t); t \to p] + c_2 [f_2(t); t \to p]$ **10**

b) IF Laplace transform of f(t) is $\overline{f}(p)$, then show that Laplace transform of $e^{at} f(t)$ is $\overline{f}(p-a)$ **10**

OR

c) If $L[f(t); t \to p] = \overline{f}(p)$, then show that $L[f(at); t \to p] = \frac{1}{a}\overline{f}\left(\frac{p}{a}\right)$. **10**

d) Evaluate: **10**

i) L[sinat]

ii) $L[f(t)]$, where $f(t) = \begin{cases} t/a, & 0 < t < a \\ 1, & t > a \end{cases}$

UNIT – IV

4. a) If $f_1(r)$ and $f_2(r)$ be two integrable functions in $(0, \infty)$ and c_1, c_2 are constants, then show that $H_n[c_1 f_1(r) + c_2 f_2(r)] = c_1 H_n[f_1(r)] + c_2 H_n[f_2(r)]$ **10**

b) If a is a constant and $H[f(r); \xi]$ is Hilbert transform of f(r) of order ν, then show that $H_\nu[f(ar); \xi] = \frac{1}{a^2} H_\nu\left[f(r); \frac{\xi}{a}\right] = \frac{1}{a^2}\overline{f}_\nu\left(\frac{\xi}{a}\right)$. **10**

OR

c) Find Hankel transform of order zero of **10**

i) $f(x)=\begin{cases}1, & 0<x>a\\ 0, & x>a\end{cases}$

ii) $\left[\dfrac{d^2}{dx^2}+\dfrac{1}{x}\dfrac{d}{dx}\right]\left(\dfrac{e^{-ax}}{x}\right)$

d) Evaluate **10**

a) $H_1\left[\dfrac{e^{-ax}}{x};\,\xi\right]$

b) $H_0\left[\dfrac{1}{x};\,\xi\right]$

5. a) Find Fourier transform of **5**

$$f(x)=\begin{cases}x, & |x|<a\\ 0, & |x|>a\end{cases}$$

b) Prove that, if f(x) is defined in (0, a) then $\overline{f}_s[1;n]=\dfrac{a}{n\pi}[1+(-1)^{n+1}]$ **5**

c) Find the Laplace transform of e^{qn} by using definition of L.T. **5**

d) Find $M[f'(x);x\to s]=-(s-1)f^*(s-1)$ **5**

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