B.Sc. - I (NEP) - Semester-I

BSCMA501 - Core DSC - Mathematics : Differential Calculus

P. Pages : 3 **GUG/W/24/15924**

Time : Three Hours Max. Marks : 80

* 7 2 4 8 *

Notes : 1. Solve all the **five** questions.
2. First four questions carry equal 15 marks and the fifth question carry 20 marks.

UNIT – I

1. Solve **any three.**

a) Let f(x) and g(x) be defined at all points of an interval [a, b] except possibly at $x_0 \in [a,b]$. If $\lim_{x \to x_0} f(x) = A$, $\lim_{x \to x_0} g(x) = B$ then prove that- **5**

$$\lim_{x \to x_0} \{f(x) + g(x)\} = \lim_{x \to x_0} f(x) + \lim_{x \to x_0} g(x) = A + B.$$

b) If **5**

$$f(x) = \frac{e^{1/x}}{1+e^{1/x}} \quad , \; x \neq 0$$
$$= 0 \quad , \; x = 0$$

Show that f(x) has a simple discontinuity at x = 0.

c) Verify the Rolle's theorem for the function $f(x) = x^2 + x - 6$ in $[-3, 2]$. **5**

d) Show that $\frac{x}{1+x^2} < \tan^{-1} x < x, \; \forall x > 0$. **5**

e) Verify Cauchy mean value theorem for $f(x) = x^2, \; g(x) = x^3$ in $[1, 3]$. **5**

UNIT – II

2. Solve **any three.**

a) Write Taylor's series for $f(x) = (1-x)^{5/2}$ with Lagrange's form of remainder upto three terms in the interval [0,1] **5**

b) Obtain Maclaurin's series for f(x) = log (1+x). **5**

c) Expand $f(x) = 2x^3 + 7x^2 + x - 1$ in powers of (x-2). **5**

d) Prove that if limit of a function $f(x,y)$ as $(x,y) \to (x_0, y_0)$ exists, then it is unique. **5**

e) Let **5**

$$f(x,y) = \frac{x^3}{x^2+y^2} \quad , \text{ for } (x,y) \neq (0,0)$$
$$= 0 \quad , \text{ for } (x,y) = (0,0)$$

Prove that f is continuous at (0,0).

UNIT – III

3. Solve **any three.**

a) Prove that if $y = e^{ax}\sin(bx+c)$ then $y_n = r^n e^{ax}\sin(bx+c+n\theta)$, **5**

Where $r = \sqrt{a^2+b^2}, \theta = \tan^{-1}\left(\frac{b}{a}\right)$.

b) Find the nth differential coefficient of $\frac{1}{1-5x+6x^2}$. **5**

c) If $y^3 + 3ax^2 + x^3 = 0$ then show that $y^5.y_2 + 2a^2x^2 = 0$. **5**

d) Find nth differential coefficient of : **5**

i) x^2e^{ax} ii) $e^x \log x$.

e) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$ then prove that- **5**

$$x^2y_{n+2} + (2n+1)xy_{n+1} + 2n^2y_n = 0.$$

UNIT – IV

4. Solve **any three.**

a) If $u = x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right); x, y \neq 0$ then prove that $\frac{\partial^2 u}{\partial x\, \partial y} = \frac{x^2-y^2}{x^2+y^2}$. **5**

b) If z = f(x, y) and $x = r\cos\theta, y = r\sin\theta$ then show that – **5**

$$\left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2 = \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$$

c) If z = f(x, y) has differential at (x_0, y_0) then f is continuous at (x_0, y_0). **5**

d) Verify Euler's theorem on homogeneous function for $u = 3x^2yz + 5xy^2z + 4z^4$. **5**

e) Find by using Lagrange's method of multipliers, the least distance of the origin from the plane x – 2y+2z = 9. **5**

5. Solve **any ten.**

a) Write $\in - \delta$ definition of limit of a function. **2**

b) Evaluate $\lim\limits_{x \to 0} \dfrac{\sin 2x}{x}$ **2**

c) State the Lagrange's Mean value theorem. **2**

d) State the Taylor's theorem for f(x + h). **2**

e) Define continuity of a function f(x, y) at (x_0, y_0). **2**

f) Using algebra of limits, prove that $\lim\limits_{(x,y)\to(1,2)} \left(x^2 + xy - 2x - y\right) = -1$. **2**

g) If $y = e^{-2x}$, find y_{11}. **2**

h) If $y = \dfrac{1}{1+x}$ then prove that $y_n = (-1)^n n!(1+x)^{-n-1}$. **2**

i) State the Leibnitz theorem. **2**

j) State Euler's theorem for homogeneous function f(x, y). **2**

k) If $z = f\left(x^2 - y^2\right)$, show that $yz_x + xz_y = 0$. **2**

l) Find the stationary points of $x^3 + y^3 - 3axy$. **2**

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