M.Sc. (Mathematics) (NEP Pattern) Semester-I

NEP-64-1 / DSE1 Elective - Numerical Analysis

P. Pages : 2 **GUG/W/24/15115**

Time : Three Hours Max. Marks : 80

Notes : 1. Solve all the **five** questions.
2. Each questions carry equal marks.

UNIT – I

1. a) Discuss about Newton's method. **8**

b) Assume f(x), $f'(x)$ and $f''(x)$ are continuous for all x in some neighbourhood of α and assume $f(\alpha)=0, f'(\alpha)\neq 0$ Then if x_0 is chosen sufficiently close to α, the iterate $x_n,\ n\geq 0$. Will converges to α. **8**

OR

c) Discuss Muller's method for finding roots of the polynomials. Discuss why Muller's method is better than the secant method. **8**

d) Assume that α is root of x = g(x) and that g(x) is P times continuously differentiable for all x near α, for some $p\geq 2$. Furthermore assume. $g'(\alpha) = ------- = g^{(p-1)}(\alpha)=0$ Then prove that if the initial guess x_0 is chosen sufficiently close to α, the iteration $x_{n+1}=g(x_n), n\geq 0$. Will have order of convergence p and **8**

$$\lim_{n\to\infty} \frac{\alpha - x_{n+1}}{(\alpha - x_n)^p} = (-1)^{p-1} \frac{g^{(p)}(\alpha)}{p!}$$

UNIT – II

2. a) Let $x_0, x_1, ..., x_n$ be distinct real numbers, and let f be a given real valued function with n + 1 continuous derivative on the interval $I_t = H\{t,\ x_0,\ x_1, ..., x_n\}$, with t some given real number then prove there exist $\xi \in I_t$ with **8**

$$f(t) - \sum_{j=0}^{n} f(x_j)\ I_j(t) = \frac{(t-x_0).......(t-x_n)}{(n+1)!} f^{(n+1)}(\xi).$$

b) Prove that $k\geq 0$ **8**

$$f[x_0,\ x_1, ..., x_k] = \frac{1}{k!\,h^k} \Delta^k f_0$$

Where, $f_0 = f(x_0)$ and $f_i = f(x_i)$.

OR

c) Find out Hermite interpelating polynomial for which. **8**

$$p(a) = f(a),\ p'(a) = f'(a)$$
$$p(b) = f(b),\ p'(b) = f'(b)$$

d) For any two functions f and g, and for any two constant α and β, Prove that **8**

$\Delta^r(\alpha f(x) + \beta g(x)) = \alpha\Delta^r f(x) + \beta\Delta^r g(x),\ r \ge 0$.

UNIT – III

3. a) Let f(x) be a continuous for $a \le x \le b$ and let $\varepsilon > 0$. Then these is a polynomial P(x) for which $|f(x) - p(x)| \le \varepsilon \qquad a \le x \le b$. **8**

b) Assuming [a, b] is finite, prove that **8**

$$\lim_{n\to\infty} \| f - r_n^* \|_2 = 0$$

OR

c) Find the error of approximating e^x using the third-degree Taylor polynomial $p_3(x)$ on the interval [-1, 1], expanding about x = 0. **8**

d) Prove that for $f, g \in C[a, b]$, **8**

$$|(f, g)| \le \| f \|_2 \ \| g \|_2$$

UNIT – IV

4. a) Derive Newton-Cotes integration formula for n = 1. **8**

b) Evaluate $I = \int_0^\pi e^x \cos x \ dx$ by Trapezoidal rule. **8**

OR

c) Discuss the simple Trapezoidal Rule. **8**

d) Obtain Peano kernel error formula. **8**

5. a) Write the newton algorithm. **4**

b) Obtain the expression for $p_1(x)$ by Lagrange interpolation **4**

c) For $f, g \in C[a, b]$ then prove that $\| f + g \| \le \| f \|_2 + \| g \|_2$. **4**

d) Define asymptotic error estimate. **4**

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