B.E. Mechanical Engineering (Model Curriculum) Semester - III

BSC-202 - Mathematics-III : (PDE, Probability & Statistics)

P. Pages : 3 **GUG/S/23/14056**

Time : Three Hours * 2 7 9 3 * Max. Marks : 80

Notes : 1. All questions carry equal marks.
2. Use of non-programmable calculator is permitted.

1. a) Solve: **8**

$$Px(z-2y^2)=(z-qy)(z-y^2-2x^3)$$

b) Solve: **8**

$$(D^2+2DD'-8D'^2)z=e^{2x+y}+\sqrt{2x+3y}$$

OR

2. a) Find the value of $\oint_C \frac{(12z-7)}{(z-1)^2(2z+3)}dz$, where C is a circle. **8**

(i) $|z|=2$ (ii) $|z+i|=\sqrt{3}$ by using Cauchy's Residue theorem.

b) Find the Laurent's series expansion of the function $F(z)=(z^2+4z+3)^{-1}$ in the region **8**

a) $1<|z|<3$ b) $0<|1+z|<2$

c) $|z|<1$ d) $|z|>3$

3. a) A discrete random variables X has the following probability function. **8**

X	0	1	2	3	4	5	6	7
F (x)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

Find:

i) K ii) P (1 < X)

iii) P (X > 6) iv) The distribution function

b) The joint probability function of two discrete random variables X & Y is given by **8**

$$F(x,y)=\begin{cases} cxy, & x=1,2,3 \ \& \ y=1,2,3 \\ 0, & \text{otherwise} \end{cases}$$

Find:

i) Constant C ii) $P(1\le x\le 2,\ y\le 3)$

iii) Find marginal probability function of X & Y.

iv) Determine whether X & Y are independent.

OR

4. a) A random variable X has density function given by **8**

$$F(x)=\begin{cases}\dfrac{1}{(b-a)}, & a<x<b\\ 0, & \text{otherwise}\end{cases}$$

Find:

i) E(X) ii) Var(X)

iii) The moment generating function iv) The first four moment about the origin.

b) A random variable X has the density function given by **8**

$$F(x)=\begin{cases}c(1-x^2), & -1\le x\le 1\\ 0, & \text{otherwise}\end{cases}$$

Find:

i) C ii) Coefficient of skewness

iii) Coefficient of Kurtosis

5. a) Find the modal matrix B corresponding to matrix $A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$ & verify that $B^{-1}AB$ is diagonal form. **8**

b) Verify Cayley – Hamilton theorem for given matrix. **8**

$$A=\begin{bmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{bmatrix}$$

& hence find the matrix represented by $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$

OR

6. a) Solve $\dfrac{d^2y}{dt^2}-3\dfrac{dy}{dt}-10y=0$ given that $y(0)=3, y'(0)=15$ by matrix method. **8**

b) Reduce the quadratic form $3x_1^2+3x_2^2+3x_3^2+2x_1x_2+2x_1x_3-2x_2x_3$ to canonical form by an Orthogonal Transformation. **8**

7. a) Obtain Fourier Series for **8**

$$F(x)=\begin{cases}-\sin\left(\pi x/L\right), & -L<x<0\\ \sin\left(\pi x/L\right), & 0<x<L\end{cases}$$

Hence show that

$$\frac{1}{2}=\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\text{..........}\infty$$

b) Solve the integral equation **8**

$$\int_0^\infty F(x).\cos\alpha x\,dx = \begin{cases} 1-\alpha, & 0 \le \alpha \le 1 \\ 0 \quad , & \alpha > 1 \end{cases}$$

& hence evaluate $\int_0^\infty \frac{\sin^2 t}{t^2} dt$.

OR

8. a) Find $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$ by using convolution theorem. **8**

b) Solve $\frac{d^2x}{dt^2} + 9x = \cos 2t$, given that $x(0) = 1,\ x\left(\pi/2\right) = -1$. **8**

9. a) Find the real root of $x\log_{10} x - 1.2 = 0$ by Newton – Raphson method correct upto Four decimal places. **8**

b) Solve: **8**

$x + 7y - 3z = -22,\ 5x - 2y + 3z = 18,\ 2x - y + 6z = 22$ by using Gauss – Seidal method.

OR

10. a) Solve $\frac{dy}{dx} = \frac{1}{2}(1+x^2)\,y^2$ & $y(0) = 1, y(0.1) = 1.06, y(0.2) = 1.12, y(0.3) = 1.21$. **8**

Find $y(0.4)$ & $y(0.5)$ by using Milne’s predictor – corrector method.

b) Solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ given that $y = 1$ when $x = 0$ & $h = 0.2$. Find y_1 & y_2 by using Runge – Kutta method. **8**

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