

M.C.A.- I (Two Years Programme) (New CBCS Pattern) Semester - I  
**PSMCAT104.2 - Elective-II Paper-IV : Discrete Mathematics**

P. Pages : 2

Time : Three Hours



GUG/S/23/13639

Max. Marks : 80

- Notes :
1. All questions are compulsory and carry equal marks.
  2. Draw neat and labelled diagrams wherever necessary.
  3. Avoid vague answers and write answers relevant and specific to questions only.

**Either:**

1. a) Show that 8
- i)  $[(P \wedge Q) \rightarrow P] \wedge [P \rightarrow (P \wedge Q) \Rightarrow (\neg P \vee Q)]$
  - ii) Show that  $(\neg P \vee Q) \vee (\neg P \wedge Q) \vee P$  is a tautology [Do not use truth table]
- b) Verify the equivalence,  $\neg A \Leftrightarrow A^* (\neg P_i)$  where,  $A(P, Q, R)$  is  $\neg P \wedge \neg(Q \wedge R)$  8

**OR**

- c) Prove by mathematical Induction. 8
- $$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$
- d) Suppose, A, B, C are matrices then prove that 8
- i)  $A(B+C) = AB+AC$
  - ii)  $(AB)C = A(BC)$

**Either:**

2. a) Let  $A = \{a, b, c, d, e\}$  & 8
- $$R = \{(a, a), (a, b), (b, c), (c, e), (c, d), (d, e)\}$$
- Compute (a)  $R^2$       b)  $R^\infty$
- b) Let  $A = \mathbb{Z}^+$ , the set of positive integers and let  $R = \{(a, b) \in A \times A \mid a \text{ divides } b\}$  is R symmetric, asymmetric or antisymmetric? 8

**OR**

- c) Let  $A = \{4, 6, 8, 10\}$  and  $R = \{(4, 4), (4, 10), (6, 6), (6, 8), (8, 10)\}$  is a relation on set A. 8
- Determine the transitive closure of R using Warshall's algorithm.
- d) If  ${}^n P_r = {}^n P_{r+1}$  and  ${}^n C_r = {}^n C_{r-1}$  find value of n and r. 8

**Either:**

3. a) What do you mean by Hasse Diagram? Determine the Hasse diagram of the relation R on set A. 8

$A = \{1, 2, 3, 4\}$  and  $R = \{(1,1), (1,2), (2,2), (2,4), (1,3), (3,3), (3,4), (1,4), (4,4)\}$

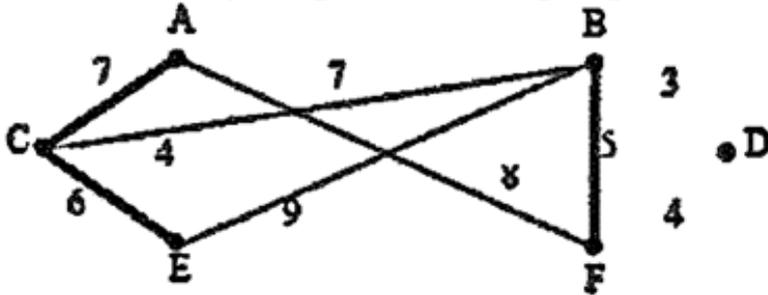
- b) Show that, in a lattice, if  $a \leq b \leq c$  then 8

i)  $(a \oplus b) = (b * c)$

ii)  $(a * b) \oplus (b * c) = b = (a \oplus b) * (a \oplus c)$

**OR**

- c) Find a minimum spanning tree of following weighted connected graph. 8



- d) Consider a lattice  $(L, *, \oplus)$  and  $a, b, \in L, a \leq b$  then set  $\{x \in L \mid a \leq x \leq b\}$  becomes a sub lattice. 8

**Either:**

4. a) Consider algebraic system  $(\{0,1\})$  where  $*$  is a multiplication operation Determine. 8

- b) Show that  $(ab)^{-1} = b^{-1}a^{-1}$  for all  $a, b, \in G$  8

**OR**

- c) Explain the following terms: 8

i) Identity

ii) Inverse

iii) Idempotent

iv) Distributivity

- d) Let T be set of all even integers. Show that the semigroup  $(z, +)$  and  $(T, +)$  are isomorphic. 8

5. Attempt all the questions.

- a) Prove that  $(P \rightarrow Q) \leftrightarrow (\neg P \rightarrow \neg Q)$  is a tautology. 4

- b) Write a short note on Relations & Digraphs with example. 4

- c) Let  $V = \{1, 2, 3, 4\}$  and  $E = \{(1,2), (1,4), (3,4), (2,3)\}$ . Draw the graph 4

- d) Find left Linear and Right Linear grammar of the following. 4

i)  $0*(1(0+1))*$

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