B.E. Computer Science & Engineering (Model Curriculum) Semester - III

SE101CS - Applied Mathematics-III

P. Pages : 3 | GUG/S/23/13801

Time : Three Hours | Max. Marks : 80

* 2 5 3 8 *

Notes : 1. All questions carry equal marks.
2. Use of non – programmable calculator is permitted.

1. a) Find $L\left\{\frac{\sin^2 t}{t}\right\}$ & hence evaluate $\int_0^\infty e^{-t}\left[\frac{\sin^2 t}{t}\right]dt$. **8**

b) Find $L\left\{t^3 \sin t\right\}$ & hence evaluate $\int_0^\infty t^3 e^{-t} \sin t . dt$. **8**

OR

2. a) Express **8**

$$F(t)=\begin{cases}\cos t \ , & 0<t<\pi \\ \cos 2t \ , & \pi<t<2\pi \\ \cos 3t \ , & t>2\pi\end{cases}$$

In terms of unit step function & hence find its Laplace Transform.

b) Find Laplace Transform **8**

$$F(t)=\begin{cases}\sin pt \ , & 0<t<\pi/p \\ 0 \ , & \frac{\pi}{p}<t<\frac{2\pi}{p}\end{cases} \quad \&$$

$$F(t)=F\left(t+\frac{2\pi}{p}\right)$$

3. a) Find $L^{-1}\left\{\frac{s}{(s^4+4a^4)}\right\}$ by using partial functions method. **8**

b) Solve $\frac{d^2x}{dt^2}+9x=\cos 2t$ given that $x(0)=1,\ x\left(\pi/2\right)=-1$. **8**

OR

4. a) Find $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$ by using convolution theorem. **8**

b) A mechanical system with two degrees of freedom satisfies the equations **8**

$$2\frac{d^2x}{dt^2}+3\frac{dy}{dt}=4,\ 2\frac{d^2y}{dt^2}-3\frac{dx}{dt}=0$$

Using Laplace Transform find x & y at any instant, given that x, y, $\frac{dx}{dt},\frac{dy}{dt}$ are zero when t = 0.

5. a) Using Fourier integral, show that **8**

$$\int_0^\infty \frac{\sin\pi\lambda\cdot\sin\lambda x}{1-\lambda^2}\,d\lambda=\begin{cases}\frac{\pi}{2}\cdot\sin x, & 0\le x\le\pi\\ 0 & , \quad x>\pi\end{cases}$$

b) Find the Fourier Transform of $F(x)=\begin{cases}1-x^2, & |x|\le 1\\ 0 & , \quad |x|>1\end{cases}$ & hence **8**

find $\int_0^\infty\left(\frac{\sin x-x\cos x}{x^3}\right)\cos\left(\frac{x}{2}\right)dx$

OR

6. a) Find Fourier sine Transform of $e^{-|x|}$ & hence show that **8**

$$\int_0^\infty \frac{x\sin mx}{(1+x^2)}dx=\frac{\pi}{2}e^{-m}, m>0$$

b) Use Fourier Integral to show that **8**

$$e^{-x}-e^{-2x}=\frac{6}{\pi}\int_0^\infty\frac{\lambda\sin\lambda x}{(\lambda^2+1)(\lambda^2+4)}d\lambda, x>0$$

7. a) Find a matrix B which reduces **8**

$A=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}$ to a diagonal form by Transformation $B^{-1}AB$.

Hence find diagonal form of A.

b) Verify Cayley – Hamilton theorem for the matrix **8**

$A=\begin{bmatrix}2&1&1\\0&1&0\\1&1&2\end{bmatrix}$ & hence find the matrix represented by

$$A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I.$$

OR

8. a) Show that **8**

$$e^A = e^x \begin{bmatrix} \cosh x & \sinh x \\ \sinh x & \cosh x \end{bmatrix} \text{ where } A = \begin{bmatrix} x & x \\ x & x \end{bmatrix}$$ By using Sylvester's Theorem.

b) Solve $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} - 10y = 0$ given that $y(0) = 3,\ y'(0) = 15$ by using matrix method. **8**

9. a) Let X be a random variable having density function **8**

$$F(x) = \begin{cases} cx, & 0 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

Find:

i) The constant c

ii) $P\left(\frac{1}{2} < X < \frac{3}{2}\right)$

iii) $P(X > 1)$

iv) The distribution function.

b) Let X & Y be continuous random variables having joint density function **8**

$$F(x, y) = \begin{cases} c(x^2 + y^2), & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Find:

i) Constant c

ii) $P\left(\frac{1}{4} < X < \frac{3}{4}\right)$

iii) Find marginal distribution function of X & Y.

iv) Determine whether X & Y are independent.

OR

10. a) A random variable X has density function given by **8**

$$F(x) = \begin{cases} 2e^{-2x}, & x \ge 0 \\ 0, & x < 0 \end{cases}$$

Find:

i) E (X)

ii) Var (X)

iii) Moment generating function

iv) The first four moment about origin.

b) A random variable X has the density function given by **8**

$$F(x) = \begin{cases} e^{-x}, & x \ge 0 \\ 0, & \text{otherwise} \end{cases}$$

Find the coefficient of

i) Skewness &

ii) Kurtosis

****