B.E. / B.Tech. (Model Curriculum) Semester - I
BSC103 - Engineering Mathematics-I

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Notes : 1. All questions carry equal marks.
2. Use of non programmable calculator is permitted.

1. a) Find the nth derivatives of $\dfrac{1}{x^2 + a^2}$ **8**

b) If $y = \tan^{-1} x$ prove that $(1+x^2)y_{n+2} + 2(n+1)x\,y_{n+1} + n(n+1)y_n = 0$. Hence find $y_n(0)$. **8**

OR

2. a) Expand $\log \sin x$ in power of $(x-2)$. **6**

b) Using Taylor's theorem, find the value of $\cos 64°$ correct to four decimal places. **6**

c) Solve $\displaystyle\lim_{x \to n/4} \dfrac{\sec^2 x - 2\tan x}{1 + \cos 4x}$ **4**

3. a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ show that $\left(\dfrac{\partial}{\partial x} + \dfrac{\partial}{\partial y} + \dfrac{\partial}{\partial z}\right)^2 \mu = -\dfrac{9}{(x+y+z)^2}$ **8**

b) If $u = \tan^{-1}\left(\dfrac{y^2}{x}\right)$ show that $x^2 \dfrac{\partial^2 u}{\partial x^2} + 2xy \dfrac{\partial^2 u}{\partial x\,\partial y} + y^2 \dfrac{\partial^2 u}{\partial y^2} = -\sin 2u \sin^2 u$. **8**

OR

4. a) If $Z = x\,f(y/x) + g(y/x)$ show that $x^2 \dfrac{\partial^2 z}{\partial x^2} + 2xy \dfrac{\partial^2 z}{\partial x\,\partial y} + y^2 \dfrac{\partial^2 z}{\partial y^2} = 0$ **8**

b) If $u = u(x, y)$ and $x = e^r \cos\theta$ $y = e^r \sin\theta$ show that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = e^{-2r}\left(\dfrac{\partial^2 u}{\partial r^2} + \dfrac{\partial^2 u}{\partial \theta^2}\right)$ **8**

5. a) If $x = r\sin\theta\cos\phi$ $y = r\sin\theta\sin\phi$ $y = r\cos\theta$ show that $\dfrac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin\theta$ **8**

b) Expand $e^x \sin y$ in power of x and y as far as terms of the third degree. **8**

OR

6. a) If $u = x + 2y + z$, $V = x - 2y + 3z$, $W = 2xy - xz + 4yz - 2z^2$ **8**
Show that they are functionally related and find the relation between them.

b) Use Lagrange's method to determine the distance from the origin to the plane   **8**
$3x + 2y + z = 12$.

7. a) Show that $\displaystyle\int_0^\infty e^{-kx} x^{n-1} dx = \dfrac{\overline{|n}}{k^n}$ **4**

b) Evaluate $\displaystyle\int_0^2 x(8 - x^3)^{1/3} dx$ **4**

c) Evaluate the integral $\displaystyle\int_0^1 \dfrac{x^\alpha - 1}{\log x} dx$ by applying differentiation under the integral sign   **8**

$(\alpha \geq 0)$.

OR

8. a) Evaluate $\displaystyle\int_0^\infty \dfrac{\tan^{-1} ax}{x(1 + x^2)} dx$ by applying differentiation under the integral sign.   **8**

b) Obtain the real mean square value of $f(t) = 3\sin 2t + 4\cos 2t$ over the range $0 \leq t \leq \pi$.   **8**

9. a) Fit a parabola $y = a + bx + cx^2$ to the following data :   **8**

x	1	2	3	4	5	6	7
y	2.3	5.2	9.7	16.5	29.4	35.5	54.4

b) Two lines of regression are given by $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$ if $\sigma_x^2 = 12$ find   **8**
i)   the mean value of x and y
ii)   the standard deviation of y
iii) the coefficient of correlation between x and y.

OR

10. a) Express $x^3 - x + 1$ in the factorial notation form.   **4**

b) Prove that $\Delta \log f(x) = \log\left[1 + \dfrac{\Delta f(x)}{f(x)}\right]$ **4**

c) Obtain the missing figure in the following table.   **8**

x	0	1	3	4	6
f(x)	4.3315	4.7046	5.6713	-	7.1154

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