B.E. / B.Tech. (Model Curriculum) Semester - II

ESC104 / BSC104 - Engineering Mathematics-II

P. Pages : 2 **GUG/S/23/13173**

Time : Three Hours Max. Marks : 80

* 1 6 1 5 *

Notes : 1. All questions carry equal marks.
2. Use of non-programmable calculator is permitted.

1. a) Solve $\frac{dy}{dx} = -\frac{ye^y}{xe^{xy} + 2y}$ **4**

b) Solve $\frac{dy}{dx} + \frac{y}{x}\log y = \frac{y}{x^2}(\log y)^2$ **4**

c) Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \cos(e^x)$ **8**

OR

2. a) Solve $(xy^2 - e^{1/x^3})dx - x^2y\,dy = 0$ **4**

b) Solve $[\cos x \log(2y-8) + \frac{1}{x}]dx + \frac{\sin x}{y-4}\,dy = 0$ **4**

c) Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 2x\,e^x + 3e^x \cos 2x$ **8**

3. a) Solve by method of variation of parameter $\frac{d^2y}{dx^2} + a^2y = \sec ax$ **8**

b) Solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x \cos(\log x)$ **8**

OR

4. a) Solve $(2x+3)^2\frac{d^2y}{dx^2} - 2(2x+3)\frac{dy}{dx} - 12y = 6x$ **8**

b) A condenser of capacity C is discharged through an inductance L and a resistance R, in series and the charge q at time t satisfies the equation. **8**

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0$$

Given L = 0.25 Henries, R = 250 ohms, $C = 2 \times 10^{-6}$ farads and that when t = 0 the charge q is 0.002 Coulombs and the current $\frac{dq}{dt} = 0$ Obtain the value of q in terms of t.

5. a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1-x^2-y^2-z^2}} \, dx\, dy\, dz$ **8**

b) Evaluate, by changing the order of integration $\int_0^a \int_0^x \frac{\cos y}{\sqrt{(a-x)(a-y)}} \, dx\, dy$ **8**

OR

6. a) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes y + z = 4 and z = 0. **8**

b) Find the centre of gravity of the area bounded by the Parabla $y^2 = x$ and the line $x + y = 2$ **8**

7. a) Find the magnitude of tangential and normal components of the acceleration of the particle moving along the curve $\bar{r} = (t^3 - 4t)\hat{i} + (t^2 + 4t)\hat{j} + (8t^2 - 3t^3)\hat{k}$ at t = 2. **8**

b) Find the directional derivatives of $\phi = x^2 y + y^2 z$ at point (2, 2, 2) in the direction normal to surface $4x^2 y + 2z^2 = 2$ at (2, -1, 3) **8**

OR

8. a) The position vector of a point at time 't' is given by $\bar{r} = e^t \cos t\, \hat{i} + e^t \sin t\, \hat{j}$ then show that $\bar{a} = 2(\bar{v} - \bar{r})$. **6**

b) Find the value of a, b, c so that the directional derivatives of $\phi = axy^2 + byz + cz^2 x^3$ point (1, 2, -1) has a maximum magnitude 64 in direction parallel to z-axis. **6**

c) If $\bar{a} = t^3 \hat{i} - t\, \hat{j} + (2t+1)\hat{k},\ \bar{b} = 2t\, \hat{i} + \hat{j} - t\, \hat{k}$ then find $\frac{d}{dt}(\bar{a} \times \bar{b})$. **4**

9. a) Show that $\bar{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force field. Find the scalar potential. Also find the work done moving an object in this field from (1, -2, 1) to (3, 1, 4). **8**

b) Show that div $r^n \bar{r} = (n+3)r^n$ Hence show that div $\frac{\bar{r}}{r^3} = 0$. **8**

OR

10. a) Evaluate $\int_c \bar{F} \cdot d\bar{r}$ by Stoke's theorem where $\bar{F} = y^2\hat{i} + x^2\hat{j} - (x+z)\hat{k}$ and C is the boundary of the triangle with the vertices (0, 0,0) (1, 0, 0) (1, 1,0) **8**

b) By Gauss divergence theorem evaluate $\iint_s \bar{F} \cdot \bar{n}\, ds$ where S is the surface of the Sphere $x^2 + y^2 + z^2 = 16$ and $\bar{F} = 3x\hat{i} + 4y\hat{j} + 5z\hat{k}$. **8**

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