

SE202 - Probability Random Process and Numerical Method

P. Pages : 3

Time : Three Hours

**GUG/S/23/13912**

Max. Marks : 80

- Notes : 1. All questions carry equal marks.
2. Use of non-programmable calculator is permitted.

1. a) The chance that a doctor A will diagnose a disease X correctly is 60%. The chance that a patient will die by his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of doctor A, who had disease X died. What is the chance that his disease was diagnosed correctly. **8**
- b) An Urn A contains 10 white and 3 black balls, while another urn B contains 3 white and 5 black balls. Two balls are transferred from Urn B to Urn A and then a ball is drawn from Urn A What is the probability that this ball is white? **8**

OR

2. a) Find the probability of getting a total of 7 **8**
i) At least once
ii) At the most twice in the five tosses of a pair of fair dice.
- b) The average rate of phone calls received is 0.7 calls per minute at an office. Determine probability that **8**
i) There will be at least one call in a Minute.
ii) There will be at least three calls during 5 minutes.

3. a) A random variables X has the following probability function. **8**

X	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k ²	2k ²	7k ² +k

- b) A Random variable X has density function. **8**

$$f(x) = \begin{cases} Cx^2, & 1 \leq x \leq 2 \\ Cx, & 2 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

Find

- i) The constant C
ii) $p(x > 2)$, $p\left(\frac{1}{2} < x < \frac{3}{2}\right)$
iii) The distribution function of x.

OR

4. a) The joint probability function of two discrete random variable x and y is given by 8

$$p(x, y) = \begin{cases} C(2x + y) & , 0 \leq x \leq 2 \\ & 0 \leq y \leq 3 \\ 0 & , \text{otherwise} \end{cases}$$

i) Find the constant C

ii) Find

$$p(x \geq 1, y \leq 2),$$

$$p(x = 2, y = 1)$$

- b) Let x be the Random variables with the following distribution and let $y = x^2$. 8

X	-2	-1	1	2
P(x)	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Find

i) The probability distribution of y

ii) The joint probability distribution of x and y

5. a) From a Sack of fruit containing 3 oranges, 2 apples and 3 bananas, a random sample of 4 pieces of fruit is selected. If x is the number of orange and y is the number of apples in the sample find 8

i) The joint probability distribution of x and y

ii) $p(x + y \leq 2)$, $p(y = 0 / x = 2)$

- b) Let x and y be continuous random variable having joint density function 8

$$f(x, y) = \begin{cases} C(x^2 + y^2) & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find (i) Constant C

(ii) $p(\frac{1}{4} < x < \frac{3}{4})$

(iii) The marginal distribution function of x and y

(iv) Determine whether x and y are independent

OR

6. a) Find the characteristics function of x whose density function is given by 8

$$f(x) = \frac{\lambda^n}{(n-1)!} x^{n-1} e^{-\lambda x} \quad x \geq 0 \quad n \text{ is a positive integer}$$

- b) Let $f(x, y) = \begin{cases} e^{-(x+y)} & x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$ 8

Be the joint density function of x and y

i) Marginal density function of x and y

ii) Conditional density function of x given y

7. a) Prove the central limit theorem for the independent and identically distributed variables. 8

$$X_k = \begin{cases} 1 & \text{Pub } \frac{1}{2} \\ -1 & \text{Pub } \frac{1}{2} \end{cases}$$

- b) Consider a random variable ω uniformly distributed over $[0, 1]$ and the sequence $x_n(\omega)$ defined as **8**

$$x_n(\omega) = \begin{cases} \frac{1}{n} & \text{For } \omega < \frac{1}{n} \\ 0 & \text{otherwise} \end{cases}$$

Note that $p(x_n = \frac{1}{n}) = \frac{1}{n}$ and $p(x_n = 0) = 1 - \frac{1}{n}$ does the sequence converge in Probability in mean square.

OR

8. a) Verify central limit theorem for a random variable x which is binomially distributed with mean np and standard deviation \sqrt{npq} . **8**

- b) Find the probability of getting between 2 heads to 4 heads in 10 tosses of fair coin using. **8**
 i) Binomial distribution
 ii) The normal approximation to the Binomial distribution.

9. a) Two random process $\{x(t)\}$ and $\{y(t)\}$ are given by **8**

$$x(t) = A \cos(\omega t + \theta)$$

$$y(t) = A \sin(\omega t + \theta)$$

Where A and ω are constants and θ is a uniform random variables over $(0, 2\pi)$. Find the cross-correlation function $x(t)$ and $y(t)$ and verify the shift of $y(t)$ in one direction is equivalent to a shift of $x(t)$ in other direction.

- b) The random binary transmission process $\{x(t)\}$ is a WSS process with zero mean and autocorrelation function $R_x(\tau) = 1 - \frac{|\tau|}{T}$ where T is constant. Find the mean and variance of the time average of $\{x(t)\}$ over $(0, T)$ is $\{x(t)\}$ mean ergodic? **8**

OR

10. a) Calculate the power spectral density of a stationary random process for which the autocorrelation is $R(z) = Ae^{-\beta|z|}$, $A > 0$, $\beta > 0$. **8**

- b) A random process is described by $x(t) = A \sin t + B \cos t$ where A and B are independent random variables with zero means and equal standard deviation. Show that the process is stationary of the second order. **8**
