B.E. Electronics & Communication / Telecommunication Engineering / Instrumentation Engineering (Model Curriculum) Semester - III

001 / IN301 - Mathematics-III / Mathematics-III (Probability and Statistics)

P. Pages : 2

Time : Three Hours

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Max. Marks : 80

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Notes : 1. All questions carry equal marks.
2. Use of Non - programmable calculator is permitted.

1. a) If $L[f(t)] = \bar{f}(s)$ then show that $L\left[\dfrac{f(t)}{t}\right] = \int_s^\infty \bar{f}(s)ds$ and also find Laplace transform **8**

of $\dfrac{\sin 3t \cdot \cos t}{t}$

b) Express $f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \end{cases}$ in terms of unit step function and find its Laplace **8**

transform.

OR

2. a) If $L[f(t)] = \bar{f}(s)$ then show that $L[t^n f(t)] = (-1)^n \dfrac{d^n}{ds^n} \bar{f}(s)$ and also find **8**

$L[t^2 e^{-t} \cos t]$

b) Find Laplace transform of f(t), **8**
where $f(t) = \begin{cases} \sin pt, & 0 < t < \pi/p \\ 0, & \pi/p < t < 2\pi/p \end{cases}$
and $f(t) = f(t + 2\pi/p)$

3. a) i) Find $L^{-1}\left\{\log\dfrac{(s+a)}{(s+b)}\right\}$ ii) Find $L^{-1}\left\{\cot^{-1}\dfrac{s}{2}\right\}$ **8**

b) By convolution theorem, find the inverse Laplace transform of $\dfrac{s^2}{\left(s^2+a^2\right)\left(s^2+b^2\right)}$ **8**

OR

4. a) Find inverse Laplace transform of $\dfrac{2s^2-6s+5}{s^3-6s^2+11s-6}$ by partial traction method. **8**

b) Solve $\dfrac{d^2x}{dt^2} + 9x = \cos 2t$, given $x(0) = 1$, $x(\pi/2) = -1$ by Laplace transform method. **8**

5. a) Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} a^2 - x^2, & \text{if } |x| < a \\ 0, & \text{if } |x| > a > 0 \end{cases}$, hence show **8**

that $\int_0^\infty \left(\dfrac{\sin x - x \cos x}{x^3}\right) dx = \pi/4$

b) Using Parseval's identity, prove that $\displaystyle\int_0^\infty \frac{t^2}{\left(t^2+1\right)^2}\, dt = \frac{\pi}{4}$ **8**

OR

6. a) Find Fourier sine transform of   **8**

$$f(x) = \frac{e^{-ax}}{x},\ a>0$$

b) Using Fourier integral show that $\displaystyle\int_0^\infty \frac{\sin \pi\lambda \sin \lambda x}{1-\lambda^2}\, d\lambda = \begin{cases} \pi/2\sin x, & 0 \le x < \pi \\ 0 & , \quad x > \pi \end{cases}$ **8**

7. a) Form the partial differential equation from the equation by eliminating the arbitrary constants.   **8**

i) $\displaystyle 2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ ii) $\displaystyle \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

b) Solve $z\left(z^2 + xy\right)\left(px - qy\right) = x^4$ **8**

OR

8. a) Solve $(x+2z)p + (4xz - y)q = 2x^2 + y$ **8**

b) Solve $\dfrac{\partial u}{\partial x} = 4\dfrac{\partial u}{\partial y}$, given that $u(0,y) = 8e^{-3y}$ by using method of separation of variable.   **8**

9. a) Find the inverse of matrix A by partitioning, where $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ **8**

b) Using Sylvester's theorem to verify $\log_e e^A = A$ where $A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$ **8**

OR

10. a) Find the characteristics equation of the matrix A, show that it is satisfied by A and hence find A^{-1} where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ **8**

b) Solve $\dfrac{d^2y}{dt^2} - 5\dfrac{dy}{dt} + 6y = 0$, given $y(0) = 2\ y'(0) = 5$ by matrix method.   **8**

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