

M.C.A.- I (Two Years Programme) (New CBCS Pattern) Semester - I
PSMCAT104.2 - Elective-II Paper-IV : Discrete Mathematics

P. Pages : 2

Time : Three Hours



GUG/S/23/13639

Max. Marks : 80

- Notes :
1. All questions are compulsory and carry equal marks.
 2. Draw neat and labelled diagrams wherever necessary.
 3. Avoid vague answers and write answers relevant and specific to questions only.

Either:

1. a) Show that 8
- i) $[(P \wedge Q) \rightarrow P] \wedge [P \rightarrow (P \wedge Q) \Rightarrow (\neg P \vee Q)]$
 - ii) Show that $(\neg P \vee Q) \vee (\neg P \wedge Q) \vee P$ is a tautology [Do not use truth table]
- b) Verify the equivalence, $\neg A \Leftrightarrow A^* (\neg P_i)$ where, $A(P, Q, R)$ is $\neg P \wedge \neg(Q \wedge R)$ 8

OR

- c) Prove by mathematical Induction. 8
- $$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$
- d) Suppose, A, B, C are matrices then prove that 8
- i) $A(B+C) = AB+AC$
 - ii) $(AB)C = A(BC)$

Either:

2. a) Let $A = \{a, b, c, d, e\}$ & 8
- $$R = \{(a, a), (a, b), (b, c), (c, e), (c, d), (d, e)\}$$
- Compute (a) R^2 b) R^∞
- b) Let $A = \mathbb{Z}^+$, the set of positive integers and let $R = \{(a, b) \in A \times A \mid a \text{ divides } b\}$ is R 8
- symmetric, asymmetric or antisymmetric?

OR

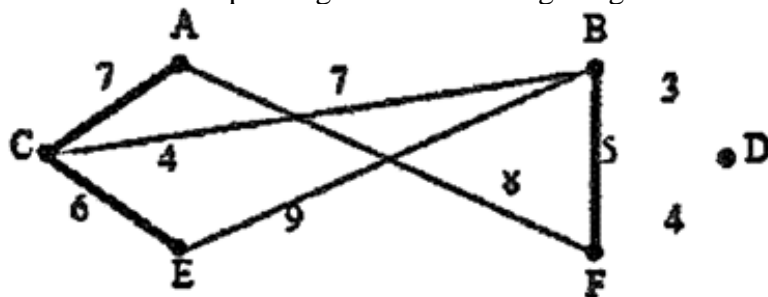
- c) Let $A = \{4, 6, 8, 10\}$ and $R = \{(4, 4), (4, 10), (6, 6), (6, 8), (8, 10)\}$ is a relation on set A. 8
- Determine the transitive closure of R using Warshall's algorithm.
- d) If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$ find value of n and r. 8

Either:

3. a) What do you mean by Hasse Diagram? Determine the Hasse diagram of the relation R on set A. 8
 $A = \{1, 2, 3, 4\}$ and $R = \{(1,1), (1,2), (2,2), (2,4), (1,3), (3,3), (3,4), (1,4), (4,4)\}$
- b) Show that, in a lattice, if $a \leq b \leq c$ then 8
 i) $(a \oplus b) = (b * c)$
 ii) $(a * b) \oplus (b * c) = b = (a \oplus b) * (a \oplus c)$

OR

- c) Find a minimum spanning tree of following weighted connected graph. 8



- d) Consider a lattice $(L, *, \oplus)$ and $a, b \in L, a \leq b$ then set $\{x \in L \mid a \leq x \leq b\}$ becomes a sub lattice. 8

Either:

4. a) Consider algebraic system $(\{0,1\})$ where $*$ is a multiplication operation Determine. 8
 b) Show that $(ab)^{-1} = b^{-1}a^{-1}$ for all $a, b \in G$ 8

OR

- c) Explain the following terms: 8
 i) Identity ii) Inverse
 iii) Idempotent iv) Distributivity
- d) Let T be set of all even integers. Show that the semigroup $(\mathbb{Z}, +)$ and $(T, +)$ are isomorphic. 8

5. Attempt all the questions.

- a) Prove that $(P \rightarrow Q) \leftrightarrow (\neg P \rightarrow \neg Q)$ is a tautology. 4
 b) Write a short note on Relations & Digraphs with example. 4
 c) Let $V = \{1, 2, 3, 4\}$ and $E = \{(1,2), (1,4), (3,4), (2,3)\}$. Draw the graph 4
 d) Find left Linear and Right Linear grammar of the following. 4
 i) $0 * (1(0+1)) *$
