

M. Tech. Electronics & Communication Engineering (CBCS Pattern) Sem-I
PECS11 - Probability Theory and Stochastic Processes

P. Pages : 2

Time : Three Hours



GUG/W/22/10978

Max. Marks : 70

- Notes :
1. Questions carry marks as indicated.
 2. Answer **five** questions from **eight** questions.
 3. Assume suitable data wherever necessary.
 4. Illustrate your answers wherever necessary with the help of neat sketches.

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1. a) Let X be the random variable giving the number of heads in three tosses of a fair coin. Find probability function and $E(X)$. **6**
- b) A random variable X has the density function **8**
$$f(x) = ke^{-3x}, \quad x > 0$$
$$= 0, \quad x \leq 0$$

Find constant k , mean and variance of X .

2. a) Six dice are thrown 729 times. How many times do you expect at least 3 dice to show 5 or 6? **6**

b) A customer knows that on average 4% of parts delivered by a manufacturer are defective and has accepted this percentage. To check whether manufacturer exceeds the limit, the customer takes from each batch of 800 parts randomly a sample of size 80 and accepts the delivery if there are at most 3 defective parts in a batch. Using hypergeometric distribution, find the probability that the customer accepts a batch which contains 50 defective parts? **8**

3. a) If 3% of electric bulbs manufactured by a company are defective find the probability that in a sample of 100 bulbs, (i) exactly 2 (ii) at most 2 (iii) at least 2 bulbs are defective. **6**

b) Show that the limiting form of the Binomial Distribution as $n \rightarrow \infty$ and $p \rightarrow 0$ in such a way that $np = \lambda$, where λ is a fixed positive number, is the Poisson Distribution. **8**

4. A person writes 'n' letters and addresses 'n' envelopes, then one letter is placed into each envelope. What is the probability that at least one letter will reach its correct destination? What if $n \rightarrow \infty$? **14**

5. a) Over a period of 12 hours, 180 calls are made at random what is the probability that in a four – hour interval, the number of calls is between 50 to 70? **6**

b) The probability of hitting a target is 0.001 for each shot. How many shots should be fired so that the probability of hitting with two or more shots is above 0.95? **8**

6. a) Suppose that $X(t)$ is normal process with **6**
 $\eta(t) = 3; C(t_1, t_2) = 4e^{-0.2|t_1 - t_2|}$

Find the probability that : (i) $X(5) \leq 2$, (ii) $|X(8) - X(5)| \leq 1$

- b) The process $X(t)$ is SSS iff the joint density $f(a, b)$ of the random variables a and b has circular symmetry, that is, if $f(a, b) = f(\sqrt{a^2 + b^2})$ prove. **8**
7. a) If X is normal random variable with mean 3 and S.D. 9, find the probability that X lies between 2 and 5. **7**
- b) What is the expected sum of the numbers that are obtained in 16 tosses of fair die? **7**
8. a) Find the power spectral density of a random sequence $X[n]$ whose auto correlation function is given by $R_{xx}[m] = a^{|m|}$. **7**
- b) If $X \sim U[-\pi, \pi]$, find the distribution of random variable $y = \cos x$, where U means uniform distribution. **7**
