

USMT-06 - Mathematics-II Paper-VI : Set Theory and Laplace Transform

P. Pages : 2

Time : Three Hours

**GUG/W/22/11613**

Max. Marks : 60

- Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT - I

1. a) Prove that 6
 $A \times (B \cup C) = (A \times B) \cup (A \times C)$

- b) If $a, b \in \mathbb{R}$ then prove that 6
 i) $a(-b) = -(ab)$
 ii) $(-a)b = -(ab)$
 iii) $(-a)(-b) = ab$

OR

- c) Prove that every infinite subset of countable set is countable. 6
 d) Let x & y be positive numbers then show that 6
 $x < y \Leftrightarrow x^2 < y^2$

UNIT - II

2. a) For $U = \{1, 2, 3, 4, 5\}$ 6
 $\tilde{A} = \frac{0.1}{1} + \frac{0.3}{2} + \frac{1}{5}$ and
 $\tilde{B} = \frac{0.4}{2} + \frac{0.2}{3}$, find $\tilde{A} + \tilde{B}$

- b) Let $A, B \in \tilde{p}(U)$ Then 6
 $\alpha \leq \beta \Rightarrow {}^\beta A \subseteq {}^\alpha A$ and
 ${}^{\beta+} A \subseteq {}^{\alpha+} A \forall \alpha, \beta \in [0, 1]$

OR

- c) Let $\tilde{A}, \tilde{B} \in p(U)$ Then prove that 6
 $\tilde{A} \subseteq \tilde{B} \Leftrightarrow {}^{\alpha+} \tilde{A} \subseteq {}^{\alpha+} \tilde{B}$
 d) Let $\tilde{A} \in p(U)$ then 6
 ${}^\alpha (A') = \left[(1-\alpha)^+ A \right]', \forall \alpha \in [0, 1].$

UNIT - III

3. a) Find the Laplace transform of 6
 $3t^2 - 2e^t - \sinh 3t + 5 \cos 4t$.

- b) If $L[f(t)] = F(s)$. Then prove that $L[e^{at}f(t)] = F(s-a)$, hence find $L[e^{-t} \cosh 2t]$. 6

OR

- c) Find the Laplace transform of $\sin t \cos 2t \cos 3t$. 6

- d) Find Laplace transform of 6

$$\cosh t \int_0^t e^t \cosh t \, dt.$$

UNIT - IV

4. a) Find the inverse Laplace transform of 6

i) $\frac{(s^2-1)^2}{s^5}$ ii) $\frac{s+3}{s^2+1}$

- b) Find the inverse Laplace transform of $\frac{1}{s(s^2+4)}$ by convolution theorem. 6

OR

- c) Solve 6

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 3te^{-t} \quad y(0) = 4, \quad y'(0) = 2.$$

- d) Find the inverse Laplace transform of 6

$$\frac{s^2-6}{s^3+4s^2+3s}$$

5. Solve any six.

- a) Let relation R defined in the set of natural numbers x is given by 2

$$R = \{(a, b)/a, b \in \mathbb{N}, a + 5b = 11\}$$

Then determine R

- b) Prove that 2

$$xy = zy \Rightarrow x = z \quad \forall x, y, z \in \mathbb{R} \quad y \neq 0.$$

- c) Define Intersection of two fuzzy set. 2

- d) Define convex fuzzy set. 2

- e) Find $L(1)$. 2

- f) Define Laplace Transform. 2

- g) State convolution theorem. 2

- h) Find $L^{-1}\left(\frac{4}{s^2+9}\right)$ 2
