

B.Tech. (Model Curriculum) Sem-I  
**BSC103 - Engineering Mathematics – I**

P. Pages : 2

Time : Three Hours



**GUG/W/22/13166**

Max. Marks : 80

- Notes : 1. All questions carry equal marks.  
2. Use of non-programable calculator is allowed.

1. a) If  $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{b}\right)^n$  then show that  $x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$  8

b) If  $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$  then show that 8  
 $y_n = (-1)^{n-1} (n-1)! \sin^n \theta \sin n\theta$  where  $\theta = \tan^{-1}\left(\frac{1}{x}\right)$ .

**OR**

2. a) Using Taylor's series find value of  $64^\circ$  correct upto four decimal places. 4

b) Evaluate  $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x}\right)^{1/x}$ . 4

c) If  $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$  then show that 8  
 $(1-x^2)y_{n+2} - (2n+3)xy_{n+1} - (n+1)^2 y_n = 0$

Hence show that

$$\frac{\sin^{-1} x}{\sqrt{1-x^2}} = x + \frac{2}{3}x^3 + \frac{8}{15}x^5 + \dots$$

3. a) If  $\theta = t^n e^{\frac{-r^2}{4t}}$  satisfies  $\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \theta}{\partial r} \right) = \frac{\partial \theta}{\partial t}$  then find the value of n. 8

b) If  $u = \sin^{-1} \sqrt{x^2 + y^2}$  then show that  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \tan^3 u$  8

**OR**

4. a) If  $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$  then show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$  8

b) If  $z = x^5 f_1\left(\frac{y}{x}\right) + y^{-5} f_2\left(\frac{x}{y}\right)$  then show that 8  
 $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} + x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 25z$

5. a) If  $y_1 = \frac{x_2 x_3}{x_1}, y_2 = \frac{x_1 x_3}{x_2}, y_3 = \frac{x_1 x_2}{x_3}$  then show that  $\frac{\partial(x_1, x_2, x_3)}{\partial(y_1, y_2, y_3)} = \frac{1}{4}$ . 8
- b) Find volume of greatest rectangular parallelopiped that can be inscribed in the ellipsoid  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$  8

OR

6. a) If  $u = \tan^{-1} x + \tan^{-1} y, v = \frac{x+y}{1-xy}$  then show that u & v are functionally related. Also find relation between them. 8
- b) The temperature T at any point (x, y, z) in space is given by  $T = 400xyz^2$ . Find the maximum & minimum temperature on surface of unit sphere  $x^2 + y^2 + z^2 = 1$ . 8
7. a) Evaluate  $\int_0^1 \frac{1}{\sqrt{-\log x}} dx$ . 4
- b) Evaluate  $\int_0^{\pi/2} \sqrt{\tan \theta} d\theta$ . 4
- c) By differentiating under Integral sign show that  $\int_0^\infty e^{-x} \left( \frac{1 - \cos ax}{x} \right) dx = \frac{1}{2} \log(1 + a^2)$  8

OR

8. a) Find root mean square value of function  $f(t) = a \sin pt + b \cos qt$  over  $(0, 2\pi)$ . 8
- b) Evaluate by differentiation under integral sign  $\int_0^\infty \frac{e^{-ax} \sin x}{x} dx$ . 8
9. a) Find two missing terms in following data 8
- |   |   |    |    |     |    |
|---|---|----|----|-----|----|
| x | 1 | 3  | 4  | 10  | 12 |
| y | 3 | -- | 18 | 102 | -- |
- b) Determine function whose first order forward difference is  $x^3 + 3x^2 + 5x + 12$ . 8

OR

10. a) Fit a parabola  $y = a + bx + cx^2$  to data. 8
- |   |     |     |     |      |      |      |      |
|---|-----|-----|-----|------|------|------|------|
| x | 1   | 2   | 3   | 4    | 5    | 6    | 7    |
| y | 2.3 | 5.2 | 9.7 | 16.5 | 29.5 | 35.5 | 54.4 |
- b) Find coefficient of correlation between variables x & y and hence find equations of regression lines from data. 8

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

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