B.Sc. (CBCS Pattern) Sem-V

USMT09 - Mathematics Paper-I - DSE : Linear Algebra

P. Pages : 2 **GUG/W/22/13115**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all five questions.
2. Each questions carry equal marks.

UNIT – I

1. a) Let U, W be subspaces of a vector space V(F). Prove that $U \bigcup W$ is a subspace of V iff $U \subseteq W$ or $W \subseteq U$. **6**

b) Prove that intersection of two subspaces of a vector space is a subspace. **6**

OR

c) Let $v_1, v_2, ---, v_n$ be n vectors of a vector space $V(F)$. **6**
Prove that
i) $[v_1, v_2, ---, v_n] = [\alpha_1 v_1, \alpha_2 v_2, ---, \alpha_n v_n], \alpha_i (\neq 0) \in F, \forall i$
ii) $[v_1, v_2] = [v_1 - v_2, v_1 + v_2]$

d) Let U, W be subspaces of an n – dimensional vector space V and dim U = dim W = n – 1, $U \neq W$. Prove that $\dim(U \bigcap W) = n - 2$. **6**

UNIT – II

2. a) Let U and V be vector spaces over the same field F. Then prove that a function $T : U \to V$ is linear iff **6**
$T(\alpha u + \beta v) = \alpha T(u) + \beta T(v), \forall \alpha, \beta \in F$ and $u, v \in V$.

b) Let a mapping $T : V_2 \to V_2$ be defined by $T(x, y) = (x', y')$, where $x' = x\cos\theta - y\sin\theta$, $y' = x\sin\theta + y\cos\theta$. Show that T is a linear map. **6**

OR

c) Let $T : U \to V$ be a linear map. Then prove that **6**
i) $R(T)$ is a subspace of V ii) $N(T)$ is a subspace of U

d) Prove that a linear map $T : V_3 \to V_3$ defined by $T(e_1) = e_1 - e_2, T(e_2) = 2e_2 + e_3$, $T(e_3) = e_1 + e_2 + e_3$ is neither 1 - 1 nor onto. **6**

UNIT – III

3. a) If W_1 and W_2 are subspaces of a finite dimensional vector space V. **6**
Show that $A(W_1 + W_2) = A(W_1) \bigcap A(W_2)$

b) If f and g are in V the dual of a vector space V such that $f(v) = 0$ implies $g(v) = 0$, prove that $g = \lambda F$ for some $\lambda \in F$. **6**

OR

c) Let U, V be finite dimensional complex vector spaces and $A: U \to V, B: U \to V$ be linear maps. If $\alpha \in C$, then prove that **6**

i) $(A+B)^{*} = A^{*} + B^{*}$ ii) $(\alpha A)^{*} = \bar{\alpha} A^{*}$

d) For a finite dimensional vector space V, show that V is a separating family. **6**

UNIT - IV

4. a) Let V be an inner product space over F then prove that **6**
$|(u, v)| \leq \|u\| \|v\|$

b) In an inner product space V over F prove the parallelogram law **6**
$\|u+v\|^2 + \|u-v\|^2 = 2\left(\|u\|^2 + \|v\|^2\right)$. Explain what this means geometrically in the special case $V = F^{(z)}$, where F is the real field and where the inner product is the usual dot product.

OR

c) Prove that $W^{\perp}$ is a subspace of V. **6**

d) Using Gram – Schmidt process, show that orthonormalise the set of vectors $\{(1,0,1,0),(1,1,3,0),(0,2,0,1)\}$ of V_4. **6**

5. Solve **any six**. **2**

a) Let V be a vector space over field F, then prove that

i) $\alpha 0 = 0, \alpha \in F$ ii) $0v = 0, \forall v \in V$

b) Define Linear independence. **2**

c) Show that $T: R^2 \to R^2$, defined by $T(x, y) = (x+1, y+2)$, is not a linear map. **2**

d) If T be a linear map on V_2 defined by **2**
$T(x_1, x_2) = (2x_1 + 3x_2, x_1 - x_2)$. Show that T is one - one & onto.

e) Let U and W be subspaces of V over F. Then prove that **2**
$U \subseteq W \Rightarrow A(W) \subseteq A(U)$

f) Prove that $A(W)$ is a subspace of V. **2**

g) Let V be an inner product space over F. In V define the distance $d(u, v)$ from u to v by $d(u, v) = \|u - v\|$. Prove that **2**

i) $d(u, v) \geq 0$ and $d(u, v) = 0 \Leftrightarrow u = v$

ii) $d(u, v) = d(v, u)$

h) Prove that $W \cap W^{\perp} = \{0\}$. **2**

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