B.E. Electronics & Communication / Telecommunication Engineering
(Model Curriculum) Sem-IV

SE202 : Probability Random Process and Numerical Method

P. Pages : 3 **GUG/W/22/13912**

Time : Three Hours * 3 1 4 1 * Max. Marks : 80

Notes : 1. All questions carry equal marks.
2. Assume suitable data wherever necessary.
3. Use of non-programmable calculator is permitted.

1. a) The joint probability function of two discrete random variable X and Y is given by **8**

$$f(x, y) = \begin{cases} c(2x+y), & 0 \le x \le 2, \quad 0 \le y \le 3 \\ 0, & \text{otherwise} \end{cases}$$

Find :

i) The constant C.

ii) $p(x \ge 2, y \le 2),\ p(x = 2, y = 1)$

iii) Find the marginal probability functions of both x and y.

iv) Find the conditional probability functions of y given that x = 2.

b) The Chance that a doctor A will diagnose a disease X correctly is 60%. The chance that a patient will die by his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of doctor A, who had disease X died. What is the chance that his disease was diagnosed correctly? **8**

OR

2. a) If x is a binomial random variable with mean 4 and variance 2.4, find the distribution of x. **8**

b) A typist kept a record of mistakes per day during 300 working days. **8**

Mistakes per day :	0	1	2	3	4
Number of days :	143	90	44	14	09

Fit Poisson distribution for the above data and find the theoretical frequencies.

3. a) Let x be a random variables giving the number of aces in a random draw of four cards from a pack of 52 cards. Find the probability function and the distribution function for x. **8**

b) A random variable x has density function **8**

$$f(x) = \begin{cases} kx^2, & 1 \le x \le 2 \\ kx, & 2 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

Find constant ‘k’ and the distribution function.

OR

4. a) A random variable x has density function given by **8**

$$f(x)=\begin{cases} x e^{-x}, & x>0 \\ 0, & \text{otherwise} \end{cases}$$

Find :

i) E(x)

ii) $E\left[(x-1)^2\right]$

iii) Var(x)

iv) First four moment about the origin.

b) Find moment generating function and first four moments about the origin for random variable 'x' given by **8**

$$x=\begin{cases} 1, \text{ prob } 1/2 \\ -1, \text{ prob } 1/2 \end{cases}$$

5. a) Let x and y have joint density function **8**

$$f(x,y)=\begin{cases} x+y, & 0\le x\le 1,\ 0\le y\le 1 \\ 0, & \text{otherwise} \end{cases}$$

Find conditional expectation and variance of y given x.

b) Let x and y be random variables having joint density function. **8**

$$f(x,y)=\begin{cases} e^{-(x+y)}, & x\ge 0,\ y\ge 0 \\ 0, & \text{otherwise} \end{cases}$$

Find :

i) Var(x)

ii) Var(y)

iii) σ_x

iv) σ_y

OR

6. a) An urn A contains 5 red, 3 white and 8 green marbles while urn B contains 3 red, and 5 white marbles. A fair die is tossed if 3 or 6 appears a marble is chosen from B otherwise from A. Find the probability that (i) a red marble is chosen (ii) A white marble is chosen (iii) A green marble is chosen by using Markov chains rule. **8**

b) Apply Chebyshev's theorem to calculate (i) $p(5<x<15)$ (ii) $p(1x-101\ge 3)$ for a random variable x with mean $\mu=10$ and variance $\sigma^2=4$ $(\sigma=\text{sigma})$ **8**

7. a) For a Linear system L and a random sequence x(n), the mean of the output random sequence y(n) is **8**

$E\{y(n)\}=L\{E[x(n)]\}$ as long as both sides are well defined.

b) Verify central limit theorem in the case where $x_1, x_2, \ldots.., x_n$ are independent and identically distributed with Poisson distribution. **8**

OR

8. a) Let x(n) be a martingale sequence defined on $n \ge 0$. Then for every $\in > 0$ and for any positive n, **8**

$$P\left[\max_{0 \le k \le n} |x(k)| \ge \in\right] \le E\left\{x^2(n)\right\} \big/ e^2$$

b) Verify central limit theorem for a random variable x which is Binomially distributed with mean np and standard deviation $\sqrt{npq}$. **8**

9. a) The mean square derivative of a WSS random process x(t) exists at time t if the autocorrelation $R_{xx}(\mathcal{T})$ has derivatives up to order two at $\mathcal{T} = 0$. **8**

b) Let f (w) be an integrable function that is real and non – negative that is $f(w) \ge 0$ for all w. Then there exists a stationary random process with power spectral density s(w) = f(w). In particular, if the random process is real – valued then f(w) is even. **8**

OR

10. a) A necessary and sufficient condition for f (T) to be a correlation function is that it be positive semidefinite. **8**

b) Let $x(n) \to x$ and $y(n) \to y$ in the m.s. sense with $E\left[|x|^2\right] < \infty$ and $E\left[|y|^2\right] < \infty$ then show that **8**

i) $\lim_{n \to \infty} E\left[|x(n)|^2\right] = E\left[|x|^2\right]$

ii) $\lim_{n \to \infty} E\left[x(n) \cdot y(n)\right] = E\left[xy\right]$

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