B.E. Electronics & Communication/Telecommunication Engineering / Instrumentation Engineering (Model Curriculum) Sem-III

001 / IN301: Applied Mathematics-III / Mathematics-III (Probability and Statistics)

P. Pages : 2 **GUG/W/22/13906**

Time : Three Hours Max. Marks : 80

Notes : 1. All questions carry equal marks.
2. Use of non-programmable calculator is permitted.

1. a) Find $L\left[e^{-t}t^2 \sin 2t\right]$ **4**

b) Evaluate $\int_0^\infty e^{-2t} \frac{\sin^2 t}{t} dt$ using laplace transform. **6**

c) Express $F(t) = \begin{cases} t^2; 0 < t < 2 \\ 4t; \quad t > 2 \end{cases}$ in terms of unit step function and hence find $L\{f(t)\}$. **6**

OR

2. a) If $L[f(t)] = \bar{f}(s)$ then show that $L\left\{\int_0^t f(t)dt\right\} = \frac{\bar{f}(s)}{s}$ and Also find lapalce transform of $\int_0^t e^t \frac{\sin t}{t} dt$. **8**

b) Find Laplace transform of f (t), where $f(t) = \begin{cases} \sin pt, & 0 < t < \pi/p \\ 0, & \pi/p < t < 2\pi/p \end{cases}$ and $f(t) = f\left(t + \frac{2\pi}{p}\right)$ **8**

3. a) Find inverse Laplace transform by partial fraction, $\bar{f}(s) = \frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}$. **8**

b) By using convolution theorem, find the inverse Laplace transform of $\frac{1}{s(s+1)(s+2)}$ **8**

OR

4. a) Solve the simultaneous differential equations by Laplace transform method. **8**
$\frac{dx}{dt} + 4\frac{dy}{dt} - y = 0,\ \frac{dx}{dt} + 2y = e^{-t}$, given that $x(0) = 0 = y(0)$.

b) Solve the equation by Laplace transform method. **8**
$y'' - 3y' + 2y = 4t + e^{3t}$ given that $y(0) = 1,\ y'(0) = -1$.

5. a) Expressed the function $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ as a Fourier integral and Hence evaluate $\int_0^\infty \frac{\sin\lambda \cdot \cos\lambda x}{\lambda} d\lambda$. **8**

b) Solve the integral equation. **8**

$$\int_0^\infty f(x)\cos\alpha x\, dx = \begin{cases} 1-\alpha, & 0 \le \alpha \le 1 \\ 0 \quad , & \alpha > 1 \end{cases}$$

OR

6. a) Using Fourier integral show that **8**

$$\int_0^\infty \frac{\lambda \sin \lambda x}{\lambda^2 + \beta^2} d\lambda = \frac{\pi}{2} e^{-\beta x},\ x > 0,\ \beta > 0$$

b) Evaluate the integral $\int_0^\infty \frac{dx}{\left(a^2+x^2\right)\left(b^2+x^2\right)}$ by using Parseval's Identity. **8**

7. a) Form the partial differential equations from the equations by eliminating the arbitrary constants, $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. **8**

b) Solve **8**

$$(y+zx)\frac{\partial z}{\partial x} - (x+yz)\frac{\partial z}{\partial y} = x^2 - y^2.$$

OR

8. a) Solve **8**

$$x\left(y^2 - z^2\right)\frac{\partial z}{\partial x} + y\left(z^2 - x^2\right)\frac{\partial z}{\partial y} = z\left(x^2 - y^2\right).$$

b) Use the method of Separation of variables to solve the equation. **8**

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u, \text{ given that } u(x,0) = 6e^{-3x}.$$

9. a) Find the modal matrix corresponding to the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. **8**

Also write diagonal matrix.

b) Solve by matrix method **8**

$$X'' - 5X' - 6X = 0 \text{ given } \begin{matrix} X(0) = 2, \\ X'(0) = 0 \end{matrix}$$

OR

10. a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ Hence find A^{-1}. **8**

b) If $M = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ then find $M^2 - 3M + 1$ by Sylvester's theorem. **8**

****