

B.E. Mechanical Engineering (Model Curriculum) Sem-III
BSC-202 : Mathematics-III (PDE, Probability & Statistics)

P. Pages : 3

Time : Three Hours



GUG/W/22/14056

Max. Marks : 80

- Notes : 1. All questions carry equal marks.
2. Use of non-programmable calculator is permitted.

1. a) Solve 8
$$(xy^3 - 2x^4)p + (2y^4 - x^3y)q = 9z(x^3 - y^3)$$

b) Solve 8
$$\frac{\partial^3 z}{\partial x^3} - 3 \frac{\partial^3 z}{\partial x \cdot \partial y^2} + 2 \frac{\partial^3 z}{\partial y^3} = (x + 2y)^{1/2}$$

OR

2. a) If $u - v = (x - y)(x^2 + 4xy + y^2)$ & $f(z)$ is an analytic Function of z , find $f(z)$ in terms of z & also find u & v . 8

b) Evaluate, by using Cauchy's internal formula $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$ where C is a circle 8
 $|z| = 3/2$

3. a) Let x be a random variables giving the number of access in a random draw of four cards from a pack of 52 cards. Find the probability function & the distribution function for x . 8

b) The joint probability function of two discrete random variables x & y is given by 8
$$F(x, y) = \begin{cases} cxy, & x = 1, 2, 3 \text{ \& } y = 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}$$

Find

- i) c
- ii) $P(1 \leq x \leq 2, y \leq 3)$
- iii) $P(x \geq 2)$
- iv) Find marginal probability function of x & y .
- v) Determine whether x & y are independent.

OR

4. a) Let 4
$$x = \begin{cases} 1, & \text{Prob } 1/6 \\ 2, & \text{Prob } 1/3 \\ 3, & \text{Prob } 1/2 \end{cases}$$

Find i) The mean ii) The variance

- b) Find moment generating function & first four moments about the origin for r.v.x given by 4

$$x = \begin{cases} 1, & \text{Prob } 1/2 \\ -1, & \text{Prob } 1/2 \end{cases}$$

- c) A random variable x has the density function given by 8

$$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Find the coefficient of

i) Skewness

ii) Kurtosis

5. a) Find the inverse of the matrix 8

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 3 & 2 \\ 2 & 4 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

by the method of partitioning.

- b) Find eigen values, eigen vectors & modal matrix for 8

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

OR

6. a) Solve $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} - 10y = 0$ given $y(0) = 3$, $y'(0) = 15$ by matrix method. 8

- b) Reduce the quadratic form 8

$8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 + 4x_2x_3 - 8x_1x_3$ to canonical form by an orthogonal transformation.

7. a) Obtain Fourier series for 8

$$f(x) = \begin{cases} -\sin\left(\frac{\pi x}{L}\right), & -L < x < 0 \\ \sin\left(\frac{\pi x}{L}\right), & 0 < x < L \end{cases}$$

hence show that

$$\frac{1}{2} = \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots \infty.$$

- b) Find Fourier Transform of $e^{-x^2/2}$. 8

OR

8. a) Find 8

$$L^{-1} \left\{ \frac{s}{(s^2 + a^2)^2} \right\} \text{ by using convolution theorem.}$$

b) Solve 8

$$(D^2 + 2D + 5)y = e^{-t} \cdot \sin t \text{ given that } y(0) = 0, y'(0) = 1 \text{ where } D = \frac{d}{dt}.$$

9. a) Find the real root of $3x - \cos x - 1 = 0$ by using Newton's Raphson method. 8

b) Solve 8

$$x + 7y - 3z = -22, 5x - 2y + 3z = 18, 2x - y + 6z = 22 \text{ By using Gauss-Seidel method.}$$

OR

10. a) Solve $10 \frac{dy}{dx} = x^2 + y^2$ given $y(0) = 1$ Find $y(0.2)$ By using Runge-Kutta method. 8

b) Solve 8

$$\frac{dy}{dx} = \frac{1}{2}(1 + x^2)y^2 \text{ \& } y(0) = 1, y(0.1) = 1.06, y(0.2) = 1.12, y(0.3) = 1.21 \text{ Find } y(0.4) \text{ \& } y(0.5).$$

By using Milne's Predictor-corrector method.
