B.Sc. Second Year (CBCS Pattern) Sem-III

USMT-06 - Mathematics Paper-II (Set Theory and Laplace Transform)

P. Pages : 2 **GUG/W/23/11613 (S)**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that $(A \cup B)' = A' \cap B'$ **6**

b) Show that a relation R is symmetric if and only if $R^{-1} = R$. **6**

OR

c) Let x and y be positive numbers then show that $x < y \Leftrightarrow x^2 < y^2$. **6**

d) Prove that every infinite subset of a countable set is countable. **6**

UNIT – II

2. a) Let $A, B \in \tilde{P}(U)$ then prove that $\alpha \le \beta \Rightarrow \beta_A \subseteq \alpha_A$ and $\beta^+{}_A \subseteq \alpha^+{}_A$, $\forall \alpha, B \in [0,1]$. **6**

b) For $U = \{1, 2, 3, 4, 5\}$, $\tilde{A} = \frac{0.1}{1} + \frac{0.3}{2} + \frac{1}{5}$ and $\tilde{B} = \frac{0.4}{2} + \frac{0.2}{3}$ find $\tilde{A} + \tilde{B}$. **6**

OR

c) Let $\tilde{A}, \tilde{B} \in P(U)$ then prove that for $\forall \alpha \in [0,1]$ **6**

$^{\alpha}(\tilde{A} \cup \tilde{B}) = {}^{\alpha}\tilde{A} \cup {}^{\alpha}\tilde{B})$.

d) Let $\tilde{A}, \tilde{B} \in P(U)$ then prove that for all $\alpha \in [0,1]$ **6**

$\tilde{A} \subseteq \tilde{B} \Leftrightarrow {}^{\alpha}\tilde{A} \subseteq {}^{\alpha}\tilde{B}$.

UNIT – III

3. a) Prove that, if $L[f(t)] = F(s)$ then $L[e^{at} \cdot f(t)] = F(s-a)$ Hence evaluate $L(e^{4t} t^3)$. **6**

b) Find Laplace transform of function $(2e^t \cdot \sin 4t \cdot \cos 2t)$. **6**

OR

c) Let $L[f(t)] = F(s)$ then prove that $L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} F(s), \ n = 1, 2, 3 \cdots$ **6**

d) Find the Laplace transform of the function **6**

$$f(t) = t \quad ; \; 0 < t < a$$
$$= 2a - t \quad , \; a < t < 2a$$
$$\text{where } f(t + 2a) = f(t)$$

UNIT – IV

4. a) Verify the convolution theorem for $f_1(t) = t$, $f_2(t) = \cos nt$. **6**

b) Find the inverse Laplace transform of **6**

i) $\dfrac{s+3}{s^2+1}$ ii) $\dfrac{s}{s^4 - a^4}$

OR

c) Solve $-\dfrac{d^4y}{dx^4} + \dfrac{d^2y}{dx^2} - 2y = 0$, $y = 0$, $y' = -1$, $y'' = 0$, $y''' = 1$ when $x = 0$. **6**

d) Find the solution by Laplace transform **6**

$$(D^2 + n^2)\,x = a\sin(nt + \alpha),$$
$$x = D_x = 0,$$
$$\text{at } t = 0$$

5. Solve **any six.**

a) Let R be a relation from A = {2, 4, 9} to B = {2, 3} defined by $R = \{(a, b) / a \in A,\ b \in B,\ a \text{ is divisible by } b\}$ find the relation R. **2**

b) Prove that $xy = zy \Rightarrow x = z$, $\forall x, y, z \in R$
$y \neq 0$ **2**

c) Define scaler cardinality and relative cardinality of a fuzzy set F. **2**

d) Define union of two fuzzy sets. **2**

e) Find $L[e^{-at}]$ by using definition of Laplace transform. **2**

f) Find $L[\sin(wt + \alpha)]$ **2**

g) Find $L^{-1}\left[\dfrac{(s^2 - 1)^2}{s^5}\right]$ **2**

h) If $L^{-1}[F(s)] = f(t)$ then prove that $L^{-1}[F(s - a)] = e^{at} \cdot f(t)$. **2**

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