B.Sc. - II (CBCS Pattern) Sem-III

USMT-05 - Mathematics Paper-I (Real Analysis)

P. Pages : 2 **GUG/W/23/11612 (S)**

Time : Three Hours Max. Marks : 60

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Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that a convergent sequence of a real numbers is bounded. **6**

b) Evaluate $\lim s_n$ of the following sequences. **6**

i) $s_n = \sqrt{n+a} - \sqrt{n+b},\ a \neq b$ ii) $s_n = \dfrac{2n^2 - n}{n+7}$

OR

c) Let $\langle s_n \rangle$ be a sequence such that $\lim s_n = \ell$ and $s_n \geq 0\ \ \forall n \in N$ then prove that $\ell \geq 0$. **6**

d) Show by Cauchy's first theorem on limits **6**

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \text{---} + \frac{1}{\sqrt{n^2+n}}\right] = 1.$$

UNIT – II

2. a) Test the convergence of the series $\dfrac{1}{x(x+2)} + \dfrac{1}{(x+2)(x+4)} + \dfrac{1}{(x+4)(x+6)} + \text{-----}$ **6**

$x \in R,\ x \neq 0$.

b) Prove that A geometric series $\sum_{n=1}^{\infty} x^{n-1}$ converges to $\dfrac{1}{1-x}$ for 0 < x < 1 and diverges for **6**

$x \geq 1$.

OR

c) Let $x_n \geq 0$ and $y_n \geq 0,\ \forall n$ such that $\lim_{n\to\infty} \dfrac{x_n}{y_n} = \ell,\ \ell \neq 0, \infty$. **6**

Then prove that Σx_n and Σy_n converge or diverge together.

d) Show that $\sum \dfrac{1}{(2n+1)^3}$ is convergent and $\sum \dfrac{1}{(2n-1)^{1/2}}$ is divergent. **6**

UNIT – III

3. a) Let X be an arbitrary non-empty set. **6**

Defined by $d(x,y) = \begin{cases} 0 & x = y \\ 1 & x \neq y \end{cases}$

show that d is a metric on X.

b) Show that the function $d: R \times R \to R$ defined by $d(x, y) = |x^2 - y^2| \quad \forall x, y \in R$ is a pseudo metric on R and is not metric on R. **6**

OR

c) Prove that finite intersection of open sets is open. Give counter example to show that arbitrary intersection of open sets need not be open. **6**

d) Prove that Every convergent sequence in a metric space is a Cauchy sequence. **6**

UNIT – IV

4. a) Show that Refinement of a partition P decreases upper sums. **6**

b) Let the function f be defined as $f(x) = \begin{cases} 1 & x \text{ is rational} \\ -1 & x \text{ is irrational} \end{cases}$ **6**

Show that f is not R-integrable over [0, 1] but $|f| \in R[0, 1]$.

OR

c) Let f be a function defined on [a, b] such that $|f(x)| \le M \quad \forall x \in [a, b]$ where M is a positive number. Prove that $\int_{a}^{-b} f(x)dx - \int_{a^-}^{b} f(x)dx \le 2M(b-a)$. **6**

d) Let f be continuous and non-negative on [a, b]. Then prove that $F(x) = \int_a^x f(t)dt$ is monotonic increasing as [a, b]. Furthermore $\int_a^b f(t)dt = F(b) \ge 0$ and equality holds true only for f is identically zero on [a, b]. **6**

5. Attempt **any six.**

a) Find fifth term and n^{th} term of sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$. **2**

b) State sandwich theorem on sequences. **2**

c) Test the convergence of series $\sum_{n=1}^{\infty} \left(\frac{n+1}{n+2} \right)$. **2**

d) Test the convergence of series $\sum \frac{1}{\sqrt{n}}$. **2**

e) Define neighborhood of a point in metric space. **2**

f) Define interior point. **2**

g) Prove that for any partition P. $L(P, f) \le U(P, f)$. **2**

h) Define lower and upper integral. **2**

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