B.Tech. / B.E. Instrumentation Engineering (Model Curriculum) Semester-III

301 / IN301 - Applied Mathematics - III (Probability & Statistics)

P. Pages : 2 **GUG/W/23/13906A**

Time : Three Hours *8196* Max. Marks : 80

Notes : 1. All questions carry equal marks.
2. Use of non-programmable calculator is permitted.

1. a) If $L[f(t)] = \bar{f}(s)$ then prove that **8**

i) $L[f'(t)] = s\bar{f}(s) - f(0)$

ii) $L[f''(t)] = s^2 \bar{f}(s) - s f(0) - f'(0)$ & also find the $L\left[\frac{d}{dt}\sin t\right]$.

b) Express **8**

$$f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \cos 2t, & \pi < t < 2\pi \\ \cos 3t, & t > \pi \end{cases}$$

in terms & unit step function & find its Laplace transformation.

OR

2. a) Evaluate $\int_0^\infty \frac{e^{-4t} - e^{-2t}}{t} dt$ **8**

b) Find the Laplace transformation of $f(t) = t^2,\ 0 < t < 2$ & $f(t) = f(t+2)$. **8**

3. a) Evaluate $L^{-1}\left[\frac{5s-2}{s^2(s-2)(s+1)}\right]$

b) Solve the following differential Equation using Laplace transformation method **8**
$(D^2 + 2D + 5)y = e^{-t} \sin t$, Given y (0) = 0 & $y'(0) = 1$.

OR

4. a) Using convolution theorem find **8**

$$L^{-1}\left[\frac{s}{(s^2 + a^2)^2}\right]$$

b) The co-ordinates (x, y) of a particle moving along a plane curve at any time t, are given by **8**
$\frac{dy}{dt} + 2x = \sin 2t, \frac{dx}{dt} - 2y = \cos 2t$. If at t = 0, y = 0 & x(0) = 1 then show that the particle moves along the curve $4x^2 + 4xy + 5y^2 = 4$.

5. a) Find the Fourier sine transform of $e^{-|x|}$ & hence show that **8**

$$\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m},\ m > 0$$

b) Using Fourier integral show that **8**

$$\int_0^\infty \frac{\sin \pi\lambda \cdot \sin \lambda x}{1-\lambda^2}\, d\lambda = \begin{cases} \pi/2 \sin x, & 0 \le x \le \pi \\ 0, & x > \pi \end{cases}$$

OR

6. a) Using Parseval's identity prove that $\int_0^\infty \frac{t^2}{(t^2+1)^2}\, dt = \pi/4$ **8**

b) Find the Fourier transform of **8**

$f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$. Hence find $\int_0^\infty \frac{\sin x}{x}\, dx$.

7. a) **8**

i) Derive a partial differential equation from $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.

ii) Solve $\frac{\partial^2 z}{\partial x \cdot \partial y} = \sin x \cdot \sin y$, for which $\frac{\partial z}{\partial y} = -2\sin y$, where x = 0 & z = 0 when y is an odd multiple of $\pi/2$.

b) Solve $y^2 p - xyq = x(z-2y)$ **8**

OR

8. a) Solve $(mz-ny)\frac{\partial z}{\partial x} + (nx-lz)\frac{\partial z}{\partial y} + mx - ly = 0$. **8**

b) Solve the following partial differential equation by method of separation of variables **8**

$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$, given $u(0, y) = 8e^{-3y}$.

9. a) Given $A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$, show that sin A = sin (1) A sing Sylvester's theorem. **8**

b) Verify Cayley Hamilton for the matrix **8**

$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ & hence express $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$ as a linear polynomial in A.

OR

10. a) Solve $\frac{d^2y}{dx^2} - \frac{3dy}{dx} - 10y = 0$, given $y(0) = 15$ by matrix method. **8**

b) Test the consistency & solve **8**

$$5x + 3y + 7z = 4$$
$$3x + 26y + 2z = 9$$
$$7x + 2y + 10z = 5$$

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