B.E. / B.Tech. (Model Curriculum) Semester-I

BSC103 - Engineering Mathematics-I

P. Pages : 3 **GUG/W/23/13166**

Time : Three Hours * 6 4 5 9 * Max. Marks : 80

Notes : 1. All questions carry equal marks.
2. Use of non-programmable calculator is permitted.

1. a) If $\cos^{-1}(y/b) = \log\left(\frac{x}{n}\right)^n$ then show that $x^2 y_{n+2} + (2n+1)\,x\,y_{n+1} + 2n^2 y_n = 0$. **8**

b) Using Taylor's theorem expand sin x in power of $x - \pi/2$. **5**

c) Evaluate $\lim_{x\to 1} \frac{x^x - x}{1 - x + \log x}$ **3**

OR

2. a) If $y = \sin\left(m \sin^{-1} x\right)$ then show that $\left(1-x^2\right)y_{n+2} - (2n+1)\,x y_{n+1} + \left(m^2 - n^2\right)y_n = 0$. **8**

b) Using Maclaurin's theorem show that $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$. **5**

c) Evaluate $\lim_{x\to 0} \frac{a^x - b^x}{x}$. **3**

3. a) If $u = \log\left(x^3 + y^3 + z^3 - 3xyz\right)$ then show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 \mu = \frac{-9}{(x+y+z)^2}$ **8**

b) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{-\sin u \cos 2u}{4\cos^3 u}$ **8**

OR

4. a) If $\theta = t^n e^{-r^2/ut}$ find what value of n will make $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t}$ **8**

b) If $z = f(u,v)$ $u = e^x \cos yv = e^x \sin y$ then show that **8**

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \left(u^2 + v^2\right)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial y^2}\right)$$

5. a) If $x+y+z=u,\ y+z=uv\ z=uvw$ then show that **8**

$$\frac{\partial(x,y,z)}{\partial(u,v,w)}=u^2v.$$

b) The temperature T at an point (x, y, z) in space is $T=400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2+y^2+z^2=1$. **8**

OR

6. a) If $u=\frac{x+y}{1-xy}$ $v=\tan^{-1}x+\tan^{-1}y$ find $\frac{\partial(u,v)}{\partial(x,y)}$ of $(xy\neq 1)$ state whether u and v are functionally related. If so find the relation. **8**

b) Expand $e^x \sin y$ in powers of x and y as far as terms of third degree. **8**

7. a) Show that- **4**

$$\int_0^\infty e^{-kx}x^{n-1}dx=\frac{\lceil n}{k^n}$$

b) Evaluate $\int_0^3 \frac{x^3}{\sqrt{3-x}}dx$ **4**

c) By differentiating under the integral sign evaluate the integral. **8**

$$\int_0^\infty \frac{e^{-ax}\sin x}{x}dx$$

Hence show that $\int_0^\infty \frac{\sin x}{x}dx=\frac{\pi}{2}$

OR

8. a) By differentiating under the integral sign find f(2) of- **8**

$$f(\alpha)=\int_0^\infty \frac{e^{-x}-e^{-\alpha x}}{x\sec x}dx,\ \alpha>0.$$

b) Find the root mean square value of the expression $a\sin pt+b\cos pt$ over the interval 0 to 2π. **8**

9. a) Fit a least square parabola $y=a_0+a_1x+a_2x^2$ for the data. **8**

x:	3	5	7	9	11	13
y:	2	3	4	6	5	8

b) Calculate Karl Pearson's coefficient of correlation and the equation of the lines of regression for the following data. **8**

x:	18	19	20	21	22	23	24	25	26	27
y:	17	17	18	18	18	19	19	20	21	22

OR

10. a) Find the missing value of y from the following data. **8**

x:	5	10	15	20	25	30	35
y:	7	-	12	17	-	28	34

b) Use Lagrange's interpolation formula to find f(0.5) from the following data. **8**

x:	0	1.0	1.5	2.5	3.0
y:	1.0	0.77	0.51	-0.05	0.26

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