B.Sc. CBCS Pattern Semester-III

USMT-06 - Mathematics Paper-II : Set Theory and Laplace Transform

P. Pages : 2 **GUG/W/23/11613**

Time : Three Hours * 6 5 9 4 * Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove De Morgan's law $(A \cup B)' = A' \cap B'$. **6**

b) Show that a relation R is symmetric if and only if $R^{-1} = R$. **6**

OR

c) Show that $\sqrt{2}$ is an irrational number. **6**

d) If a and b are nonzero in R then prove that $(ab)^{-1} = a^{-1}b^{-1}$. **6**

UNIT – II

2. a) Let $A = \left\{\frac{0}{a}, \frac{0}{b}, \frac{0.4}{c}, \frac{0.5}{d}, \frac{0.6}{e}, \frac{0.9}{f}\right\}$ in $U = \{a, b, c, d, e, f\}$ then find $S(A)$, scalar cardinality and relative cardinality of the fuzzy set A in U. **6**

b) Let $U = \{1, 2, 3, 4, 5\}$, $A = \frac{0.1}{1} + \frac{0.3}{2} + \frac{1}{5}$ and $B = \frac{0.4}{2} + \frac{0.2}{3}$, find $A + B$. **6**

OR

c) Let $A, B \in P(U)$ then prove that $\forall \alpha \in [0,1]$ ${}^{\alpha}(A \cup B) = {}^{\alpha}A \cup {}^{\alpha}B$. **6**

d) Find AB. Where. **6**

$$A = \frac{0.9}{1} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.3}{6}$$

$$B = \frac{0.1}{2} + \frac{0.4}{3} + \frac{0.5}{4} + \frac{0.8}{5}$$

are defined on the Universe $U = \{1, 2, 3, 4, 5, 6\}$

UNIT – III

3. a) If f(t) is piecewise continuous in every finite interval $0 \le t \le T$. For every $T > 0$ and of exponential order α for t > T, then prove that its Laplace transform F(s) exists $\forall s > \alpha$. **6**

b) Find Laplace transform of **6**

$f(t) = 3t^2 - 2e^t + \sinh 3t + 5\cos 4t$

OR

c) If $L[f(t)] = F(s)$ then prove that $L[e^{at} f(t)] = F(s-a)$ hence find Laplace transform of **6**

$e^{-3t}(2\cos 5t - 3\sin 5t)$

d) Find the Laplace transform of $2x'' + x' - x, \quad x(0) = 0 = x'(0)$ **6**

4. **UNIT – IV**

a) Obtain inverse Laplace transform of $\dfrac{s^2 + 29}{(s^2 + 4)(s^2 + 9)}$. **6**

b) Let the functions f (t) and g (t) satisfy the hypothesis of the existence theorem of their Laplace transform if $L^{-1}[F(s)] = f(t)$ and $L^{-1}[G(s)] = g(t)$ then prove that $L^{-1}[f(s) \cdot G(s)] = \int_0^t f(u) g(t-u)\,du$ **6**

OR

c) Find the inverse Laplace transform of $\dfrac{1}{s(s^2 + 4)}$ by convolution theorem. **6**

d) Solve DE by Laplace transform method. **6**
$\dfrac{d^4y}{dx^4} + \dfrac{d^2y}{dx^2} - 2y = 0, \quad y = 0, \quad y' = -1, \quad y'' = 0$ and $y''' = 1$ when $x = 0$

5. Attempt **any six**.

a) Let R be a relation from $A = \{2, 4, 9\}$ to $B = \{2, 3\}$ defined by **2**
$R = \{(a, b) \mid a \in A, b \in B, a \text{ is divisible by } b\}$
Find the relation R

b) Find power set of A = {2, 3, 5}. **2**

c) Let U be the universe of discourse and Let $A, B \in P(U)$ then prove that **2**
$^{\alpha}+A \subseteq {}^{\alpha}A, \forall \alpha \in [0,1]$

d) Define support of a fuzzy set. **2**

e) Obtain Laplace transform of e^{at} by definition. **2**

f) Prove that $L[f''(t)] = s^2F(s) - sF(0) - f'(0)$ **2**

g) Find $L^{-1}\left[\dfrac{s+1}{s^2 + 2s + 5}\right]$ **2**

h) If $L^{-1}[F(s)] = f(t)$ then $L^{-1}\left[\int_s^{\infty} F(u)\,du\right] = ?$ **2**

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