



- Notes :
1. All questions carry marks as indicated.
  2. Answer **any five** questions.
  3. Assume suitable data wherever necessary.
  4. Illustrate your answers wherever necessary with the help of neat sketches.

1. Let there be 'n' sticks each of which is broken down into one long and one short part. The '2n' parts are arranged into 'n' pairs from which new sticks are formed. Find the probability that **14**
- a) The parts will be joined in the original order
  - b) All long parts are paired with shorter parts.
2. a) If X is a discrete random variable with a Moment generating function of  $M_X(v)$ . Find **12**  
the moment generating function if
- i)  $Y = aX + b$
  - ii)  $Y = kX$
  - iii)  $Y = \frac{X + a}{b}$
- b) Find the characteristic function for the random variable X with probability density **2**  
function given by
- $$f(x) = \frac{1}{b-a}, \quad a \leq x \leq b$$
- $$= 0, \quad \text{otherwise}$$
3. a) The probability density function of a random variable X is given by **10**
- $$f(x) = \frac{x^2}{81}, \quad -3 < x < 6$$
- $$= 0, \quad \text{otherwise}$$
- Find density function of  $y = \frac{1}{3}(12 - x)$ .
- b) If  $E(X) = 2$ ,  $E(Y) = 3$ ,  $E(XY) = 10$ ,  $E(X^2) = 9$ ,  $E(Y^2) = 16$ , then find **4**  
i) variance and ii) covariance of X and Y
4. **14**  
If  $f(x) = \frac{x}{2}, \quad 0 \leq x \leq 2$   
 $= 0, \quad \text{otherwise}$   
and  $g(y) = 2(1-y), \quad 0 \leq y \leq 1$   
 $= 0, \quad \text{otherwise}$   
Determine the function  $y(x)$  which will transform  $f(x)$  into  $g(y)$
5. a) Evaluate mean and variance of exponential distribution. **7**
- b) Determine Binomial distribution for which mean is 4 and variance is 3. **7**

6. a) Suppose  $X(t)$  is normal process with  $x(t) = 3$  and  $c(t_1, t_2) = 4.e^{-0.2|t_1-t_2|}$  Find the probability that
- i)  $X(5) \leq 2$
  - ii)  $|X(8) - X(5)| \leq 1$
- b) If the life  $x$  (in years) of a certain type of bike has Weibull distribution with the parameter  $\alpha$ . Given that probability that the life of the bike exceeds 6 years is  $e^{-0.72}$  for those values of  $\alpha$  and  $\beta$ . Find mean and variance of  $x$ .
7. a) A random process defined by  $X(t) = A \cos(\omega_0 t + \theta)$  where  $A$  and  $\omega_0$  are constants,  $\theta$  is uniformly distributed random variable in the interval  $(0, 2\pi)$ . Show that  $X(t)$  is a wide sense stationary process. 8
- b) Establish necessary and sufficient condition for the stationarity of the process  $X(t) = a \cos \omega t + b \sin \omega t$ . 6
8. a) How can we differentiate Ergodic process with stationary process? Justify your answers with examples. 7
- b) Describe in brief Wiener-Levy process. 7

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