B.Sc.- II CBCS Pattern Semester-III

USMT-05 - Mathematics Paper-I : Real Analysis

P. Pages : 2 **GUG/W/23/11612**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all the **five** questions.
2. All the questions carry equal marks.

UNIT – I

1. a) Show that a convergent sequence of real numbers is bounded. Also, if $\lim x_n = x$ & limit $y_n = y \neq 0$ then show that $\lim\left(\frac{x_n}{y_n}\right) = \frac{x}{y}$ **6**

b) Show that $\lim\limits_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + ----- + \frac{1}{\sqrt{n^2+n}}\right] = 1$ **6**

OR

c) Show that the sequence $\langle s_n \rangle, s_n = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ------ + \frac{1}{n!}$ is convergent. **6**

d) Prove that every Cauchy sequence of real numbers is bounded. **6**

UNIT – II

2. a) Prove that the necessary condition for convergence of the series $\sum x_n$ is $\lim x_n = 0$. Hence show that the series $\sum\limits_{n=1}^{\infty}(-1)^n$ is either convergent or divergent **6**

b) By using Cauchy convergence criterion show that the series $\sum \frac{1}{n}$ is divergent. **6**

OR

c) Test the convergence of the series, **6**
$\frac{4}{7}x + \frac{7}{11}x^2 + \frac{10}{15}x^3 + \frac{13}{19}x^4 + ------$

d) State the Leibnitz test & test the convergence of the series **6**
$\sum\limits_{n=1}^{\infty}(-1)^{n-1}\frac{n}{n^2+1}$

UNIT – III

3. a) Show that $d(x,y)=|x-y|, \forall x,y \in R$ defines a metric on R. **6**

b) Let (X,d) be a metric space & x, y, $x', y' \in X$. Show that **6**
$|d(x,y)-d(x',y')| \le d(x,x')+d(y,y')$

OR

c) Show that every closed sphere in a metric space is a closed set. **6**

d) Prove that $\{s^n\}$ is a Cauchy sequence of real numbers iff $\{s^n\}$ is convergent in R. **6**

UNIT – IV

4. a) Show that any constant function defined on a bounded closed interval is integrable. **6**

b) Let a function defined on [a, b] such that $|f(x)| \le M \forall x \in [a,b]$ then show that **6**
$\int_{a}^{\overline{b}} f(x)dx - \int_{\underline{a}}^{b} f(x)dx \le 2M(b-a)$

OR

c) Show that if f is monotonic in [a, b] then it is integrable on [a, b]. **6**

d) If f is continuous & integrable on [a, b] then prove that **6**
$\int_{a}^{b} f(x)dx = f(c)(b-a)$
Where c is some point in [a, b].

5. Solve **any six**:

a) Give an example of two divergent sequences s & t such that their sum (s + t) converges. **2**

b) State the monotone convergence theorem. **2**

c) Define the Alternating series. **2**

d) State the D'Alembert's ratio test. **2**

e) Show that $\bar{E} \subseteq F$ if $E \subset F \& F$ is closed. **2**

f) Define bounded metric space. **2**

g) For any partition P show that. **2**
$L(p,f) \le U(p,f)$.

h) If f is continuous on [a, b] and $|f(x)| \le K, \forall x \in [a,b]$, where K is a constant then prove that $\left|\int_{a}^{b} f(x)dx\right| \le K(b-a)$ **2**

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