B.Sc. CBCS Pattern Semester-V

USMT09 - DSE - Mathematics Paper-I (Linear Algebra)

P. Pages : 3 **GUG/W/23/13115**

Time : Three Hours Max. Marks : 80

* 5 1 1 6 *

Notes : 1. Solve all five questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that, a nonempty subset U of a vector space V over field F is a subspace of V iff **6**

i) $u+v \in U, \forall u, v \in U$

ii) $\alpha u \in U \ \forall \ \alpha F, u \in U$

b) Let U and W be subspaces of a vector space V_3, **6**
where $U = \{(x_1, x_2, x_3) \in V_3 / x_1 + x_2 = x_3\}$
$W = \{(x_1, x_2, x_3) \in V_3 / x_1 = x_2 = x_3\}$
Show that V_3 is the direct sum of U & W.

OR

c) Prove that the set $B = \{(1,1,1)(1,-1,1),(0,1,1)\}$ is a basis of V_3. **6**

d) Let W be a subspace of a finite dimensional vector space V over F. **6**
Prove that there exists a subspace W_1 of V such that $V = W \oplus W_1$.

UNIT – II

2. a) Let U, V be rector spaces over a field F and $T : U \to V$ be a linear Map. **6**
Then prove that

i) $T(0) = 0$

ii) $T(-u) = -T(u) \ \forall u \in U$

iii) $T(\alpha_1 u_1 + \alpha_2 u_2 + ----+\alpha_n u_n) = \alpha_1 Tu_1 + \alpha_2 Tu_2 + ---+\alpha_n T(u_n)$
$\forall u_i \in U, \ \alpha_i \in F, i = 1, 2 ----n$

b) Let a mapping $T : V_2 \to V_2$ be defined by $T(x, y) = (x', y')$ where **6**
$x' = x\cos\theta - y\sin\theta, y' = x\sin\theta + y\cos\theta$ Show that T is a linear map.

OR

c) Let $T : U \to V$ be a linear map then prove that. **6**

i) R(T) is a subspace of V

ii) N(T) is a subspace of U.

d) Show that the linear map $T : V_3 \to V_3$ defined by **6**
$T(x_1, x_2, x_3) = (x_1 + x_2 + x_3 + x_2 + x_3 \ x_3)$ is non singular and find its inverse.

UNIT – III

3. a) Let V be the finite dimensional vector space over F then prove that. **6**
$V \approx \hat{\hat{V}}$

b) Let W_1 and W_2 are two subspaces of finite dimensional vector space V **6**
Then prove that
$A(W_1 + W_2) = A(W_1) \wedge A(W_2)$
Where $A(W_1)$ and $A(W_2)$ are annihilator of W_1, W_2

OR

c) Show that $A(A(W)) = W$. **6**

d) Show that $A(W_1 \cap W_2) = A(W_1) + A(W_2)$ **6**
Where W_1 and W_2 are suspaces of a finite dimensional vector space V over F.

UNIT – IV

4. a) Let V be an inner product space over F. If $u, v \in V$ then prove that $|(u, v)| \le \|u\| \cdot \|v\|$ **6**

b) Let $\{x_1, x_2, ----x_n\}$ be an orthogonal set then prove that **6**
$\|x_1 + x_2 + ----x_n\|^2 = \|x_1\|^2 + \|x_2\|^2 + ------+\|x_n\|^2$

OR

c) If $\{W_1, W_2 ---W_M\}$ is an orthonormal set in V then prove that **6**
$\sum_{i=1}^{m} |(W_i, v)|^2 \le \|v\|^2$ for any $v \in V$

d) Using Gram-Schmidt orthogonalization process, orthonormalize the L.I. Subset **6**
$\{(1,1,1), (0,1,1), (0,0,1)\}$ of V_3.

5. Solve **any six** questions.

a) Let V be a vector space over F then prove that $\alpha \cdot 0 = 0, \forall \alpha \in F$ **2**

b) If S and T are subsets of a vector space V then prove that. **2**
$SST \Rightarrow L(S) \subseteq L(T)$

c) Let $T: U \to V$ be a linear map then prove that T is one-one $\Leftrightarrow$ N(T) is a zero subspace of U **2**

d) Let $F: V_2 \to V_3$ be a linear map defined by $T(x_1, x_2) = (x_1 - x_2, x_2 - x_1, -x_1)$ show that T is 1-1. **2**

e) Let U and W be subspaces of vector space V over F then prove that **2**
$U \subset W \Rightarrow A(W) \subset A(U)$

f) Define Annihilator of W = A (W). **2**

g) Prove that $W \cap W^1 = \{0\}$ **2**

h) Prove that, If V is a inner product space over F then. **2**
$(u, \alpha v + BW) = \overline{\alpha}(u, v) + \overline{B}(u, w) \forall u, v, w \in V, \alpha, B \in F$

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