

**101 / 001 - Engineering Mathematics-III**

P. Pages : 2

**GUG/W/23/13906B**

Time : Three Hours



Max. Marks : 80

- Notes : 1. All questions carry equal marks.  
2. Use of non programmable calculator is permitted.

1. a) Find Laplace Transform of  $\frac{e^{-at} - e^{-bt}}{t}$ , hence evaluate  $\int_0^\infty \frac{e^{-at} - e^{-bt}}{t} dt$ . 8

b) Find Laplace Transform of  $\sin \sqrt{t}$ . 8

**OR**

2. a) Given  $L\left\{\frac{2\sqrt{t}}{\pi}\right\} = \frac{1}{s^3/2}$ , show that  $L\left\{\frac{1}{\sqrt{\pi t}}\right\} = \frac{1}{\sqrt{s}}$ . 8

b) Find the Laplace of the following. 8

i)  $t(3\sin 2t - 2\cos 2t)$                       ii)  $\int_0^t x^2 e^{-x} dx$

3. a) Find  $L^{-1}\left\{\frac{s}{s^4 + s^2 + 1}\right\}$  8

b) Find  $L^{-1}\left\{\frac{s}{(s^2 + a^2)^2}\right\}$  by using convolution theorem. 8

**OR**

4. a) Use convolution theorem to find Laplace inverse of the following function. 8  
$$\frac{1}{(s-2)(s+2)^2}$$

b) Solve the equation  $\frac{dx}{dt} - y = e^t$ ,  $\frac{dy}{dt} + x = \sin t$ , given  $x(0) = 1$ ,  $y(0) = 0$ . 8

5. a) Express  $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$  as Fourier integral and find  $\int_0^\infty \frac{\sin \lambda \cdot \cos \lambda x}{\lambda} d\lambda$  8

b) Find the Fourier transform of  $f(x) = \begin{cases} 1-x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$  and hence find 8

$$\int_0^\infty \left( \frac{\sin x - x \cos x}{x^3} \right) \cos\left(\frac{x}{2}\right) dx.$$

**OR**

6. a) Find the  $f(x)$  if its sine transform is  $\frac{e - a\lambda}{\lambda}$ . Hence deduce the inverse sine transform of  $\frac{1}{\lambda}$ . 8
- b) Prove that Parseval identities. 8
- i)  $\int_0^\infty \bar{f}_c(s) \bar{g}_c(s) ds = \int_0^\infty f(x) g(x) dx$       ii)  $\int_0^\infty \bar{f}_s(s) \bar{g}_s(s) ds = \int_0^\infty f(x) g(x) dx$

7. a) Eliminating the arbitrary constant function  $\phi$  from the relation  $\phi(x^3 - y^3, x^2 - z^2) = 0$  and find the partial differential equation. 8
- b) Solve  $(x^2 - y^2 - yz)p + (x^2 - y^2 - zx)q = z(x - y)$ . 8

**OR**

8. a) From the partial differential equation by eliminating arbitrary constant from the equation  $xyz = f(x^2 + y^2 + z^2)$  8
- b) Solve  $(x^2 - y^2 - yz)p + (x^2 - y^2 - zx)q = z(x - y)$  8

9. a) Find inverse of the following matrices by partitioning method. 8
- $$A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$
- b) Test the consistency and solve. 8
- $2x - y - z = 2, x + 2y + z = 2, 4x - 7y - 5z = 2$

**OR**

10. a) Find a matrix B which reduces  $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$  to a diagonal form by transformation  $B^{-1}AB$ . Hence find diagonal form of A 8
- b) Find the eigen value eigen vector and model matrix for the matrix. 8
- $$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

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