B.Sc. - III CBCS Pattern Semester-V

USMT10 - (DSE-II) Mathematics Paper-II (Mechanics)

P. Pages : 2 **GUG/W/23/13116**

Time : Three Hours * 5 1 1 7 * Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Find the equations of path & component of accelerations if the velocities of the particle along & perpendicular to radius vector from a fixed origin are λr^2 & $\mu\theta^2$. **6**

b) Prove that : **6**

i) $\frac{d\hat{r}}{dt} = \dot{\theta}\hat{e}$

ii) $\frac{d\theta}{dt} = -\dot{\theta}\hat{r}$

OR

c) A point moves in a curve so that its tangential & normal accelerations are equal, prove that velocity varies as e^{ψ}. Also if the angular velocity of the tangent is constant, find the curve. **6**

d) A particle is moving in a parabola with uniform angular velocity about the focus, prove that its acceleration at any point is proportional to the radius of curvature of path at that point. **6**

UNIT – II

2. a) Prove that if the forces acting on the particle are conservative the total energy of particle is conserved. **6**

b) For the system of particles show that $N = \dot{M}$ where $N^{(e)}$ is the total external torque & M is the total angular momentum. **6**

OR

c) Show that the magnitude R of the position for the center of mass from an arbitrary origin is given by. **6**

$$M^2R^2 = M\sum_i m_i r_i^{\,2} - \frac{1}{2}\sum m_i m_j r_{ij}^{\,2}$$

d) A particle moves in straight line with constant acceleration a. If at time t = 0, the particle position is x_0 & its velocity is v_0 then prove that **6**

$$v = v_0 + at,\ x = x_0 + v_0 t + \frac{1}{2}at^2 \ \&\ v^2 = v_0^2 + 2a(x - x_0)$$

UNIT – III

3. a) State & prove D'Alembert principle. **6**

b) Construct the Lagrangian for a particle moving in a space & then deduce the equation of motion. **6**

OR

c) Derive the Lagrange's equations of motion for partly conservative system. **6**

d) Prove that the rate of energy dissipation due to friction is 2R. **6**

UNIT – IV

4. a) Derive the differential equation for the orbit for a central force field. **6**

b) A particle moves on a curve $r^n = a^n \cos n\theta$ under the influence of a central force field. Find the law of force. **6**

OR

c) State & prove the Kepler's first law about the motion of the planet. **6**

d) Show that a particle moving under a central force such that $V = kr^{n+1}$ the virial theorem reduces to $\overline{2T} = (n+1)\overline{V}$. **6**

5. Solve **any six**:

a) Define the radial & transverse velocities. **2**

b) Define the curvature & radius of curvature of the curve. **2**

c) Show that $\dot{M} = 0$. **2**

d) Define the virtual work. **2**

e) Define the Rayleigh's dissipation function. **2**

f) Show that the force of constraint does no work in any possible displacement. **2**

g) State the Kepler's second law. **2**

h) State the virial theorem. **2**

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