

101 / 001 - Engineering Mathematics-III

P. Pages : 2



GUG/W/23/13906B

Time : Three Hours

Max. Marks : 80

- Notes : 1. All questions carry equal marks.
2. Use of non programmable calculator is permitted.

1. a) Find Laplace Transform of $\frac{e^{-at} - e^{-bt}}{t}$, hence evaluate $\int_0^{\infty} \frac{e^{-at} - e^{-bt}}{t} dt$. 8

b) Find Laplace Transform of $\sin \sqrt{t}$. 8

OR

2. a) Given $L\left\{\frac{2\sqrt{t}}{\pi}\right\} = \frac{1}{s^{3/2}}$, show that $L\left\{\frac{1}{\sqrt{\pi t}}\right\} = \frac{1}{\sqrt{s}}$. 8

b) Find the Laplace of the following. 8

i) $t(3\sin 2t - 2\cos 2t)$ ii) $\int_0^t x^2 e^{-x} dx$

3. a) Find $L^{-1}\left\{\frac{s}{s^4 + s^2 + 1}\right\}$ 8

b) Find $L^{-1}\left\{\frac{s}{(s^2 + a^2)^2}\right\}$ by using convolution theorem. 8

OR

4. a) Use convolution theorem to find Laplace inverse of the following function. 8

$$\frac{1}{(s-2)(s+2)^2}$$

b) Solve the equation $\frac{dx}{dt} - y = e^t, \frac{dy}{dt} + x = \sin t$, given $x(0) = 1, y(0) = 0$. 8

5. a) Express $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ as Fourier integral and find $\int_0^{\infty} \frac{\sin \lambda \cdot \cos \lambda x}{\lambda} d\lambda$ 8

b) Find the Fourier transform of $f(x) = \begin{cases} 1-x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$ and hence find 8

$$\int_0^{\infty} \left(\frac{\sin x - x \cos x}{x^3}\right) \cos\left(\frac{x}{2}\right) dx.$$

OR

6. a) Find the $f(x)$ if its sine transform is $\frac{e^{-a\lambda}}{\lambda}$. Hence deduce the inverse sine transform of $\frac{1}{\lambda}$. 8

b) Prove that Parseval identities. 8

i) $\int_0^\infty \bar{f}_c(s) \bar{g}_c(s) ds = \int_0^\infty f(x)g(x) dx$ ii) $\int_0^\infty \bar{f}_s(s) \bar{g}_s(s) ds = \int_0^\infty f(x)g(x) dx$

7. a) Eliminating the arbitrary constant function ϕ from the relation $\phi(x^3 - y^3, x^2 - z^2) = 0$ and find the partial differential equation. 8

b) Solve $(x^2 - y^2 - yz)p + (x^2 - y^2 - zx)q = z(x - y)$. 8

OR

8. a) From the partial differential equation by eliminating arbitrary constant from the equation $xyz = f(x^2 + y^2 + z^2)$ 8

b) Solve $(x^2 - y^2 - yz)p + (x^2 - y^2 - zx)q = z(x - y)$ 8

9. a) Find inverse of the following matrices by partitioning method. 8

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$

b) Test the consistency and solve. 8
 $2x - y - z = 2, x + 2y + z = 2, 4x - 7y - 5z = 2$

OR

10. a) Find a matrix B which reduces $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ to a diagonal form by transformation 8

$B^{-1}AB$. Hence find diagonal form of A

b) Find the eigen value eigen vector and model matrix for the matrix. 8

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$
