B.Sc. (Part-I) CBCS Pattern Semester-I

USMT-02 - Mathematics Paper-II (Differential Calculus and Trigonometry)

P. Pages : 2 **GUG/W/23/11557**

Time : Three Hours * 6 5 3 8 * Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. Each questions carry equal marks.

UNIT – I

1. a) Using $\varepsilon-\delta$ definition of a limit of a function prove that **6**

$$\lim_{(x,y)\to(1,1)}\left(x^2+2y\right)=3$$

b) If a real valued function f (x, y) is continuous at $P_0\left(x_0, y_0\right)$, then prove that there is neighbourhood of P_0 in which f (x, y) is bounded. **6**

OR

c) Show that $\frac{\partial^2 z}{\partial x\,\partial y}=\frac{-1}{x\ln(ex)}$ at the point x = y = z on the surface $x^x y^y z^z = k$. Where k is constant. **6**

d) If $z=f(x,y)$ and $x=r\cos\theta,\ y=r\sin\theta,$ then show that **6**

$$\left(\frac{\partial z}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial z}{\partial\theta}\right)^2=\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2$$

UNIT – II

2. a) Verify Euler's theorem on homogeneous functions for **6**

$$u(x,y)=\log\frac{x+y}{x-y}$$

b) If $x=\rho\cos\phi,\ y=\rho\sin\phi,\ z=z$ find $\frac{\partial(x,y,z)}{\partial(\rho,\phi,z)}$. **6**

OR

c) Prove that $f(x,y): x^2+2xy+y^2$ has a local minimum along the line $x+y=0$. **6**

d) Use the method of Lagrange multipliers to locate all local maxima and minima and also find the absolute maximum or minimum of **6**

$$f(x,y,z)=xy+yz+zx \text{ where } x^2+y^2+z^2=1$$

UNIT – III

3. a) Show that the subtangent at any point of the curve $x^m y^n = a^{m+n}$ varies as the abscissa. **6**

b) Find the radius of curvature at point (x, y) on the curve $y^2 = 4ax$. **6**

OR

c) Find the asymptotes of **6**
$x^3 + 3x^2 y - 4y^3 - x + y + 3 = 0$

d) Trace the curve $x^2 y^2 = a^2 \left(y^2 - x^2\right)$ **6**

UNIT – IV

4. a) Prove that **6**
$\frac{1+\sin\theta + i\cos\theta}{1+\sin\theta - i\cos\theta} = \sin\theta + i\cos\theta$. Hence prove that
$\left(1+\sin\frac{\pi}{5} + i\cos\frac{\pi}{5}\right)^5 + i\left(1+\sin\frac{\pi}{5} - i\cos\frac{\pi}{5}\right)^5 = 0$

b) Find all the values of $(1+i)^{1/4}$. **6**

OR

c) If $\tan(\theta + i\phi) = e^{i\alpha}$, show that $\theta = \left(n + \frac{1}{2}\right)\frac{\pi}{2}$ & $\phi = \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$ **6**

d) Show that **6**
$\log\frac{x+iy}{x-iy} = 2i\tan^{-1}\frac{y}{x}$

5. Solve **any six.**

a) Using algebra of limits prove that $\lim_{(x,y)\to(1,2)} \left(x^2 + xy - 2x - y\right) = -1$ **2**

b) If $z = f(x\,y)$, then show that $xz_x - yz_y = 0$. **2**

c) Write Euler's theorem for homogeneous function of two variables. **2**

d) For the following mapping, find Jacobian determinant of the mapping. **2**
$u = 2x - y,\ v = x + 4y$

e) Find the normal at (1, 3) to the curve $y = x^2 + 2$. **2**

f) Write definition of asymptote. **2**

g) Express 1 + i in polar form. **2**

h) Prove that $\sin iz = i\sinh z$. **2**

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