B.Sc. CBCS Pattern Semester-I

USMT-01 - Mathematics Paper-I (Differential and Integral Calculus)

P. Pages : 2

Time : Three Hours

GUG/W/23/11556

Max. Marks : 60

Notes : 1. Solve all the **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) If $\lim\limits_{x \to x_0} f(x)$ exists then prove that it is unique. **6**

b) Prove that $f(x) = x^2$ is continuous at x= 3 by $\in - \delta$ definition. **6**

OR

c) Prove that f(x) is differentiable at $x = x_0$ then it is continuous at x_0. **6**

d) If $y = \left(x^2 - 1\right)^n$, prove that $\left(x^2 - 1\right) y_{n+2} + 2xy_{n+1} - n(n+1) y_n = 0$ **6**

UNIT – II

2. a) If a real function f defined on [a, b] is **6**

i) Continuous on [a, b] and (ii) differentiable on (a, b). then prove that there is at least one point $C \in (a\ b)$ such that f (b) – f (a) = (b – a) $f'(c)$.

b) Verify Rolle's theorem for the function $f(x) = x^2 + x - 6$ in [-3, 2]. **6**

OR

c) Obtain Maclaurin's series for $f(x) = \sin x$. **6**

d) Expand $2x^3 + 7x^2 + x - 1$ in powers of (x – 2). **6**

UNIT – III

3. a) Prove that $\overline{|(n+1)} = n\overline{|n}$, by using definition of gamma function. **6**

b) Prove that $B(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$. **6**

OR

c) Prove that $B(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta \cdot d\theta$. **6**

d) Prove that $\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right) = 0$ **6**

UNIT – IV

4. a) Prove that **6**

i) $f(x,y) \ge 0$ on $D \Rightarrow \iint_D f(x,y)\,dA \ge 0$

ii) $f(x,y) \le g(x,y) \Rightarrow \iint_D f(x,y)\,dA \le \iint_D g(x,y)\,dA$

b) Evaluate $\int_{-2}^{2} dy \int_{y^2-1}^{3} (x+2y)\,dx$. **6**

OR

c) Evaluate $\iint_D \frac{dx\,dy}{x^4+y^2}$, ,where D is the region $x \ge 1,\ y \ge x^2$. **6**

d) Evaluate $\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y}\,dy\,dx$ by changing the order of integration. **6**

5. Attempt **any six.**

a) Evaluate by using limit theorem. **2**
$\lim_{x\to 3}\left(2x^3 - 3x^2 + 7x - 11\right)$.

b) Define limit at infinity. **2**

c) State Cauchy's mean value theorem. **2**

d) Find $c \in (0,\pi)$ if $f(x) = \frac{\sin x}{e^x}$ by using Rolle's theorem. **2**

e) Evaluate $\lim_{x\to 1}\frac{\left(1-x^x\right)}{x\log x}$ by L'Hospital's rule. **2**

f) Evaluate $B(3/2,\ 5/2)$. **2**

g) Evaluate $\int_1^2\int_1^3 xy^2\,dx\,dy$ **2**

h) Determine the limits of integration for $\iint_D f(x,y)\,dx\,dy$ where D is the triangle with vertices O (0, 0), M (4, 0), P (4, 5) **2**

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