B.Tech. / B.E. Computer Science & Engineering (Model Curriculum) Semester-III

SE101CS / 101 - Applied Mathematics-III

P. Pages : 2 **GUG/W/23/13801**

Time : Three Hours Max. Marks : 80

Notes : 1. All questions carry equal marks.
2. Use of non programmable calculator is permitted.

1. a) Find A^{-1} by partitioning method $A=\begin{bmatrix} -p & q & 0 \\ q & p & 0 \\ 0 & 0 & 1 \end{bmatrix}$ where $p^2+q^2=1$. **8**

b) Test for consistency and solve the system of equation $2x-3y+7z=5, 3x+y-3z=13,$ $2x+19y-47z=22$. **8**

OR

2. a) Find the Eigen values and Eigen vector of $A=\begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$. **8**

b) Diagonalize the given matrix $A=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. **8**

3. a) Given $A=\begin{bmatrix} 2 & -5 \\ 1 & -4 \end{bmatrix}$, using Cayley – Hamilton theorem to evaluate A^5. **8**

b) Solve $\frac{d^2y}{dt^2}-\frac{3dy}{dt}-10y=0$, given $y(0)=3, y'(0)=15$ by matrix method. **8**

OR

4. a) Find by iteration the largest eigen value of $A=\begin{bmatrix} 1 & -3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$. **8**

b) Using Sylvester theorem prove that $\log_e e^A = A$, where $A=\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$. **8**

5. a) Show that $Z^{-1}\left\{e^{2/z}\right\}=\frac{2^n}{n!}$. **8**

b) Find the inverse Z-Transform of $\frac{3z^2-18z+26}{(z-2)(z-3)(z-4)}$. **8**

OR

6. a) Using Fourier integral show that $\int_0^\infty \frac{w.\sin(xw)}{1+w^2}dw=\frac{\pi}{2}e^{-x}, x>0$. **8**

b) Find the Fourier Cosine Transform of e^{-x^2}. **8**

7. a) If random variable X is such that $E\left[(X-1)^2\right]=10,\ E\left[(X-2)^2\right]=6$, find (i) $E(X)$ (ii) $Var(X)$ and (iii) σ_X (iv) $E(X^2)$. **8**

b) A random variable X has the density function given by $f(x)=\begin{cases} e^{-x} & x \ge 0 \\ 0 & \text{otherwise} \end{cases}$, then find coefficient of (i) Skewness (ii) Kurtosis **8**

OR

8. a) Let X be a R.V. with probability function, $P(X=x)=\frac{1}{k^x};\ x=1,2,3,......$, where k is a constant then find: **8**

i) Moment Generating Function
ii) Mean
iii) Variance

b) A R.V X is defined by $X=\begin{cases} -2 & \text{prob.}\frac{1}{3} \\ 3 & \text{prob.}\frac{1}{2} \\ 1 & \text{prob.}\frac{1}{6} \end{cases}$, then **8**

Find:
i) $E(X)$ ii) $E(2X+3)$
iii) $E(X^2+5X)$ iv) $E(2X+7)+E(X^2)$

9. a) Consider the experiment of throwing two dices. Let X denotes the sum of two dices, then obtain (i) Probability mass function, (ii) Distribution function (iii) $P(1<X\le 7)$. **8**

b) Prove that for suitable constant C. **8**

$$F(X)=\begin{cases} 0 & ;\ x \le 0 \\ C\left(1-e^{-x}\right)^2 & ;\ x>0 \end{cases}$$

is the distribution function for random variable X. Find (i) C (ii) Density function (iii) Determine $P(1<x<2)$ (iv) F(2)

OR

10. a) Let X and Y be two R.V with joint density function **8**

$$f(x,y)=\begin{cases} 9e^{-3(x+y)} & ;\ x,y \ge 0 \\ 0 & ;\ \text{otherwise} \end{cases}$$

i) Check whether this is valid density function
ii) Find $P(0\le x\le 1,\ 0\le y\le 1)$ and show that X and Y are independent.

b) Let X and Y be R.V. having joint probability function. **8**

$$f(x,y)=\begin{cases} C(2x+y) & ;\ x=0,1,2,\ y=0,1,2,3 \\ 0 & ;\ \text{otherwise} \end{cases}$$

Find: (i) C (ii) Marginal Probability function of X and Y (ii) $P(X\ge 1, Y\le 2)$ (iii) $P(X=2, Y=1)$ (iv) & (y/x)

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