B.E. Electronics & Communication/Telecommunication Engineering (Model Curriculum)
Semester-IV

SE202 - Probability Random Process and Numerical Method

P. Pages : 3 **GUG/W/23/13912**

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry equal marks.
2. Use of non-programmable calculator is permitted.

1. a) In binary communication channel A is the input & B is the output. Find P(A/B) & P(A/B') if P(A) = 0.4, P(B/A) = 0.9 & P(B'/A') = 0.6. **8**

b) An urn A contains 10 white & 3 black balls, while another urn B contains 3 white & 5 black balls. Two balls are transferred from urn B to urn A & then a ball is drawn from urn A. What is the probability that this ball is white? **8**

OR

2. a) If X is a binomial random variable with mean 4 & variance 2.4. Find the distribution function of X. **8**

b) The average rate of phone calls received is 0.7 calls per minute at an office. Determine probability that **8**
i) There will be at least one call in a minute
ii) There will be at least three calls during 5 minutes.

3. a) Let X be a random variable giving the number of aces in a random draw of four cards from a pack of 52 cards. Find the probability function & the distribution function for X. **8**

b) A random variable X has density function **8**

$$f(x) = \begin{cases} kx^2 & , \quad 1 \le x \le 2 \\ kx & , \quad 2 < x < 3 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find :
i) Constant K
ii) P(1 < x < 3)
iii) P(2 < x < 3)
iv) The distribution function.

OR

4. a) Find mean, variance & moment generating function for exponential distribution. **8**

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & , \quad x > 0 \\ 0 & , \quad x \le 0 \end{cases}$$

& find first four moments about the origin.

b) Let X be a random variables having density function **8**

$$f(x) = \begin{cases} cx & , \quad 0 \le x \le 2 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find

i) The constant C ii) $P\left(\frac{1}{2} < X < 3/2\right)$

iii) $P(X > 1)$ iv) The distribution function.

5. a) The joint probability function of two discrete random variables X & Y is given by **8**

$$f(x, y) = \begin{cases} cxy & , \quad x = 1, 2, 3 \ \& \ y = 1, 2, 3 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find

i) The constant C ii) $P(1 \le x \le 2, \ y \le 3)$

iii) Find marginal probability function of X & Y

iv) Determine whether X & Y are independent.

b) Let X & Y be continuous random variable having joint density function **8**

$$f(x, y) = \begin{cases} c(x^2 + y^2) & , \quad 0 \le x \le 1, 0 \le y \le 1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find

i) Constant C ii) $P\left(\frac{1}{4} < x < 3/4\right)$

iii) The marginal distribution function of X & Y.

iv) Determine whether X & Y are independent.

OR

6. a) Let $f(x, y) = \begin{cases} e^{-(x+y)} & , \quad x \ge 0, \ y \ge 0 \\ 0 & , \quad \text{otherwise} \end{cases}$ be the joint density function of X & Y. **8**

Find

i) Marginal density functions of X & Y

ii) Conditional density function of X given Y.

b) Let X & Y be random variables having joint density function **8**

$$f(x, y) = \begin{cases} \dfrac{3x(x+y)}{5} & , \quad 0 \le x \le 1, 0 \le y \le 2 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find

i) E(X) ii) E(Y)

iii) Var(X) iv) Var(Y)

7. a) A bank teller serves customers standing in the queue one by one suppose that the service time X_i for customer i has mean $EX_i = 2$ (minutes) & $Var(X_i) = 1$. We assume that service times for different bank customers are independent. Let Y be the total time the bank teller spends serving 50 customers. Find $P(90 < y < 110)$. **8**

b) Prove central limit theorem for the independent variables. **8**

$$X_K = \begin{cases} 1 & , \text{ Prob P} \\ -1 & , \text{ Prob q} \end{cases}$$

OR

8. a) Verify central limit theorem for a random variable X which is binomially distributed with mean np & standard deviation $\sqrt{npq}$. **8**

b) Find the probability of getting between 2 heads to 4 heads in 10 tosses of fair coin using **8**
i) Binomial distribution
ii) The normal approximation to the binomial distribution.

9. a) If a random process {x(t)} for which each sample function of the process is of the form $X(t) = \sin(wt + \theta)$ where w is constant & θ is a random variables uniformly distributed over a range $0 \le \theta \le 2\pi$ prove that $\{x(t)\}$ is a wide-sense stationary process. **8**

b) Find the mean square value of the process whose power spectral density is **8**

$$\frac{w^2 + 2}{w^4 + 13w^2 + 36}$$

OR

10. a) Find the average power of the random process $\{x(t)\}$, if it's power spectral density is given by $s(w) = \dfrac{8}{(w^2 + 9)^2}$ **8**

b) If the auto covariance function of a stationary process $\{x(t)\}$ is given by $C(T) = A\,e^{-\alpha|T|}$, prove that $\{x(t)\}$ is mean-ergodic. Also find $\operatorname{Var}\{A[x(t)]\}$ where $A\{x(t)\}$ is the time average of $\{x(t)\}$ over $(-T, T)$. **8**

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