B.Tech. / B.E. Mechanical Engineering (Model Curriculum) Semester-III

ME203 / BSC202 - Engineering Mathematics-III (PDE, Probability & Statistics)

P. Pages : 3 **GUG/W/23/14056**

Time : Three Hours * 7 6 4 3 * Max. Marks : 80

Notes : 1. All questions carry equal marks.
2. Use of non programmable calculator is permitted.

1. a) Solve $z(p-q)=z^2+(x+y)^2$. **8**

b) Solve $\frac{\partial u}{\partial x}-2\frac{\partial u}{\partial y}=u$, given that $u(x,0)=3e^{-5x}+2e^{-3x}$. **8**

OR

2. a) Evaluate $\oint_c \frac{\cos \pi z^2}{(z-1)(z-2)}dz$, where C is a circle $|z+i|=1.5$. **8**

b) Find the Laurent's series expansion of the function $f(z)=\frac{1}{(z-1)(z-2)}$ in the region. **8**

i) $1<|z|<2$

ii) $|z|>2$

3. a) A random variable X has the density function **8**

$$f(x)=\begin{cases} ke^{-3x}, & x>0 \\ 0, & x\le 0 \end{cases}$$

Find-

i) Constant k,

ii) $P(x\ge 2)$

iii) $P(x<1)$

b) The joint probability function of two discrete random variables x and y is given by **8**

$$f(x,y)=\begin{cases} cxy, & x=1,2,3 \ \& \ y=1,2,3 \\ 0, & \text{otherwise} \end{cases}$$

Find
i) The constant C
ii) Find marginal probability function of x and y
iii) Determine whether x and y are independent

OR

4. a) Find the moment generating function of the random variable. **8**

$$x = \begin{cases} 1/2, & \text{prob } 1/2 \\ -1/2, & \text{prob } 1/2 \end{cases}$$

Also find first four moments about the origin.

b) A random variable X has the density function given by **8**

$$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Find the coefficient of

i) Skewness and kurtosis

5. a) **8**

Find the inverse of matrix A by partitioning, where $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

b) **8**

Use Sylvester's theorem to show that 3 tan A = (tan 3) A, where $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$

OR

6. a) Verify Cayley-Hamilton theorem for matrix. **8**

$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^{-1}

b) **8**

Solve $\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 0$, given $y(0) = 2,\ y'(0) = 5$.

7. a) Find Fourier series for $f(x) = 2x - x^2$ in the interval $0 < x < 2$. **8**

b) **8**

Express $f(x) = \begin{cases} 1, & \text{for } |x| < 1 \\ 0, & \text{for } |x| > 1 \end{cases}$ as Fourier integral and hence evaluate $\int_0^\infty \frac{\sin\lambda}{\lambda} d\lambda$.

OR

8. a) Show that **8**

$$\int_0^\infty \frac{\cos 6t - \cos 4t}{t} dt = \log\left(\frac{2}{3}\right)$$

b) **8**

Find $L^{-1}\left\{\frac{s}{\left(s^2 + a^2\right)^2}\right\}$

By convolution theorem.

9. a) Use Crout's method to solve the equations. **8**
$x + y + z = 1,\ 3x + y - 3z = 5,\ x - 2y - 5z = 10$.

b) Solve by Gauss-Seidel method. **8**
$x + 7y - 3z = -22,\ 5x - 2y + 3z = 18,\ 2x - y + 6z = 22$.

OR

10. a) Use Runge-Kutta method to find approximate value of y for x= 0.2, when $\frac{dy}{dx} = x^2 + y^2$ **8**
given $y(0) = 1,\ h = 0.1$.

b) Solve $\frac{dy}{dx} = 2e^x - y$, given that $y(0) = 2,\ y(0.1) = 2.010,\ y(0.2) = 2.040,\ y(0.3) = 2.090$. **8**
Find $y(0.4)$ and $y(0.5)$.

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